المحاضرة القادمة الثلاثاء ١٤/١٢ امتحان ٢

Chapter 4: Forces and Equilibrium of A Particle.

In this chapter we shall consider forces applied only to mas points, Or Particles, and rigid bodies.

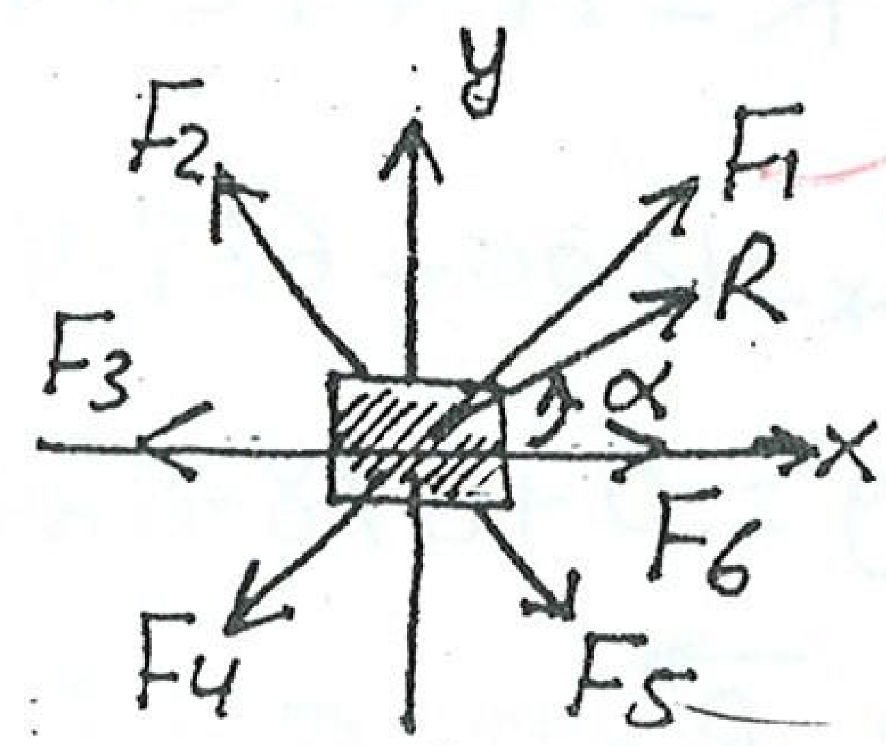
In MKSC; The unit of force is the <u>newton</u>; also we can express The force by (Kilogram-Force : kg.f) where:

$1\ Kgf = 9.8\ N$

* Composition of concurrent forces:

If the forces are concurrent (all forces applied at the Same point), their resultant is their vector sum; let The resultant $\vec{R}$ of several concurrent forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots$ is:

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots = \sum \vec{F_i}$$



If The forces are Coplanar (متحدة المستوى) (x-y):

$$\vec{R} = i R_x + j R_y$$

where: $R_x = F_{1x} + F_{2x} + F_{3x} + \ldots = \sum F_{ix}$

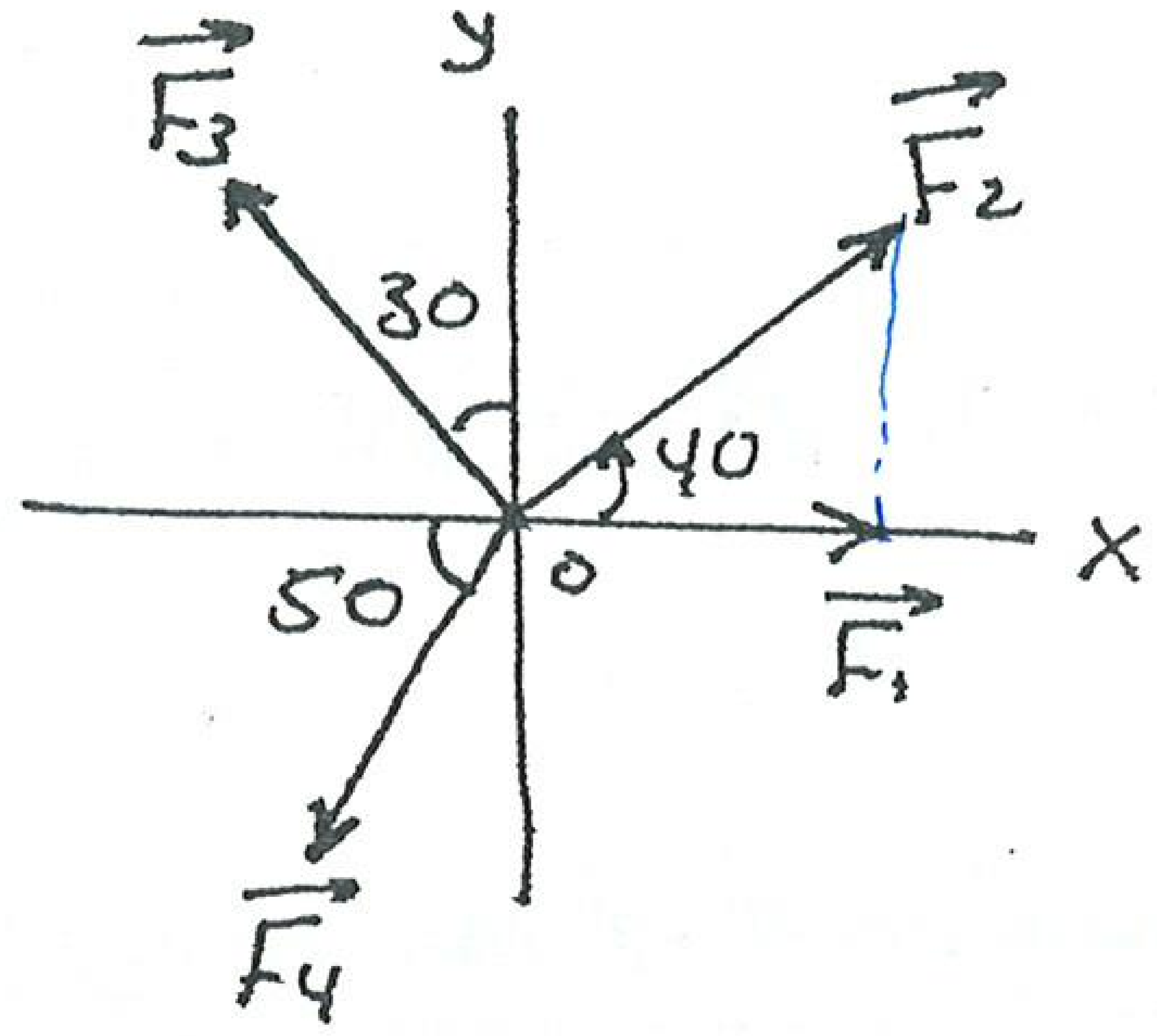
$R_y = F_{1y} + F_{2y} + F_{3y} + \ldots = \sum F_{iy}$

- magnitude of $\vec{R}$ is: $R = \sqrt{R_x^2 + R_y^2}$
- The direction of $\vec{R}$ is: $\tan\alpha = \dfrac{R_y}{R_x}$

ex: Find $\vec{R}$, of The following forces acting on body at O

$F_1 = 1200\,N$, $F_2 = 900\,N$, $F_3 = 300\,N$, $F_4 = 800\,N$

sol.



$\vec{F_1} = 1200\,i$

$\vec{F_2} = (F_2 \cos 40)\,i + (F_2 \sin 40)\,j$

$\vec{F_2} = 689.4\,i + 578.5\,j$

$\vec{F_3} = -F_3 \cos 120\,i + F_3 \sin 120\,j$

$\vec{F_3} = -150\,i + 259.8\,j$

$\vec{F_4} = -F_4 \cos 50\,i - F_4 \sin 50\,j$

$\vec{F_4} = -514.2\,i - 612.8\,j$

$\because \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$R_x = 1200 + 689.4 - 150 - 514.2 = 1225.2\,N$

$R_y = 0 + 578.5 + 259.8 - 612.8 = 225.5\,N$

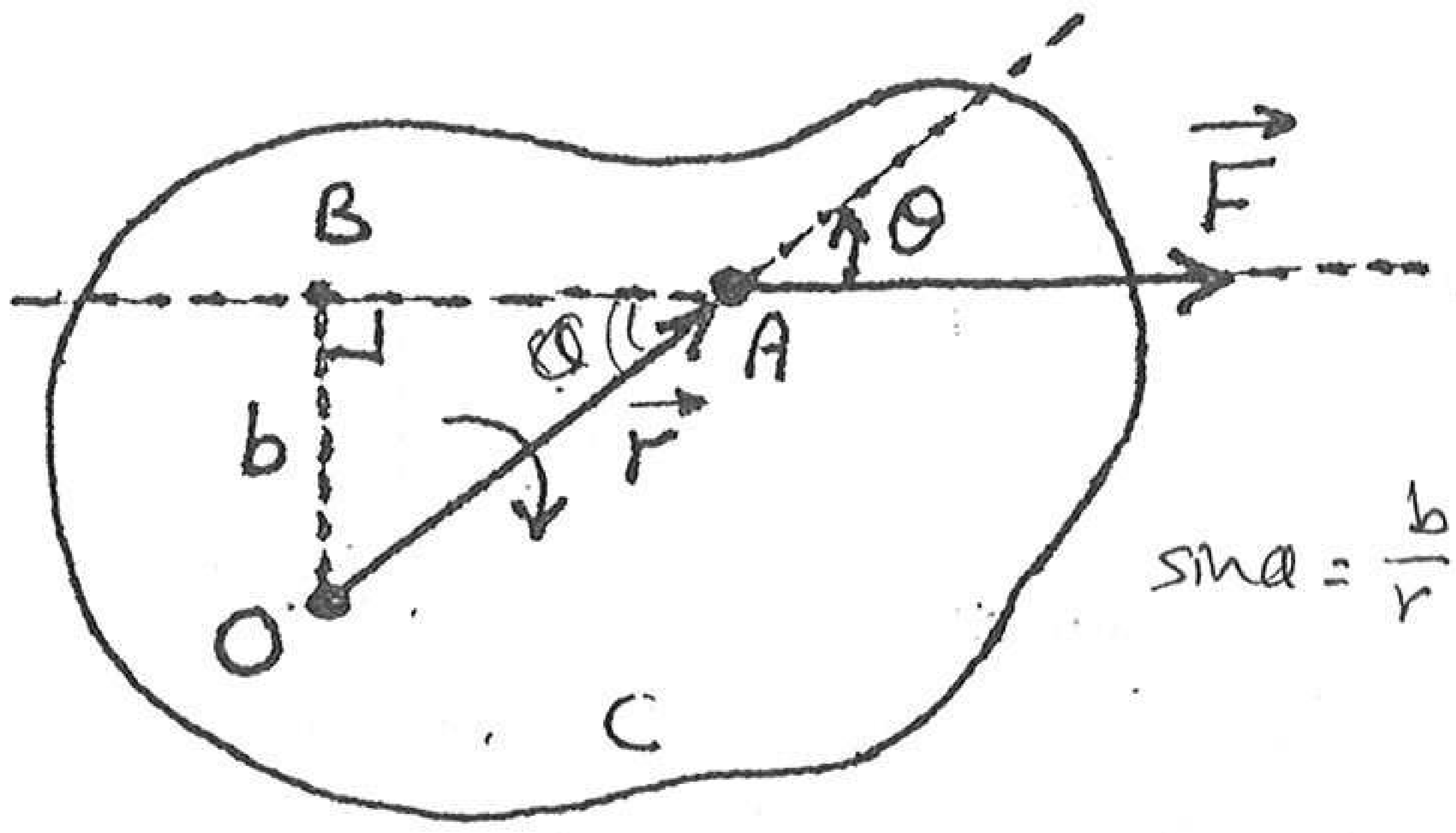
$\therefore \vec{R} = 1225.2\,i + 225.5\,j$

$R = \sqrt{(1225.2)^2 + (225.5)^2} = 1245\,N$

$\tan\alpha = \dfrac{R_y}{R_x} = \dfrac{225.5}{1225.2} \Rightarrow \alpha = 10.4^\circ$

Torque: when a Force acts on an extended (منتشر ، ممتد) body, The body does not merely (فحسب) move in the direction of the force but usually changes its Orientation (اتجاهه) by turning (بالدوران).

b: Perpendicular distance from acting force line to The Point O; also called leverarm (ذراع القوة) : $b = OB$



$\sin\alpha = \frac{b}{r}$

Torque $\tau = Fb$, $(N \cdot m)$

or torque = force X lever arm.

العزم سيكون عمودي على r, F
باستخدام قاعدة اليد اليمنى

$\because b = r\sin\theta$

يخضع للضرب الاتجاهي

$\Rightarrow \tau = F r \sin\theta$; Comparing with Cross Product:

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

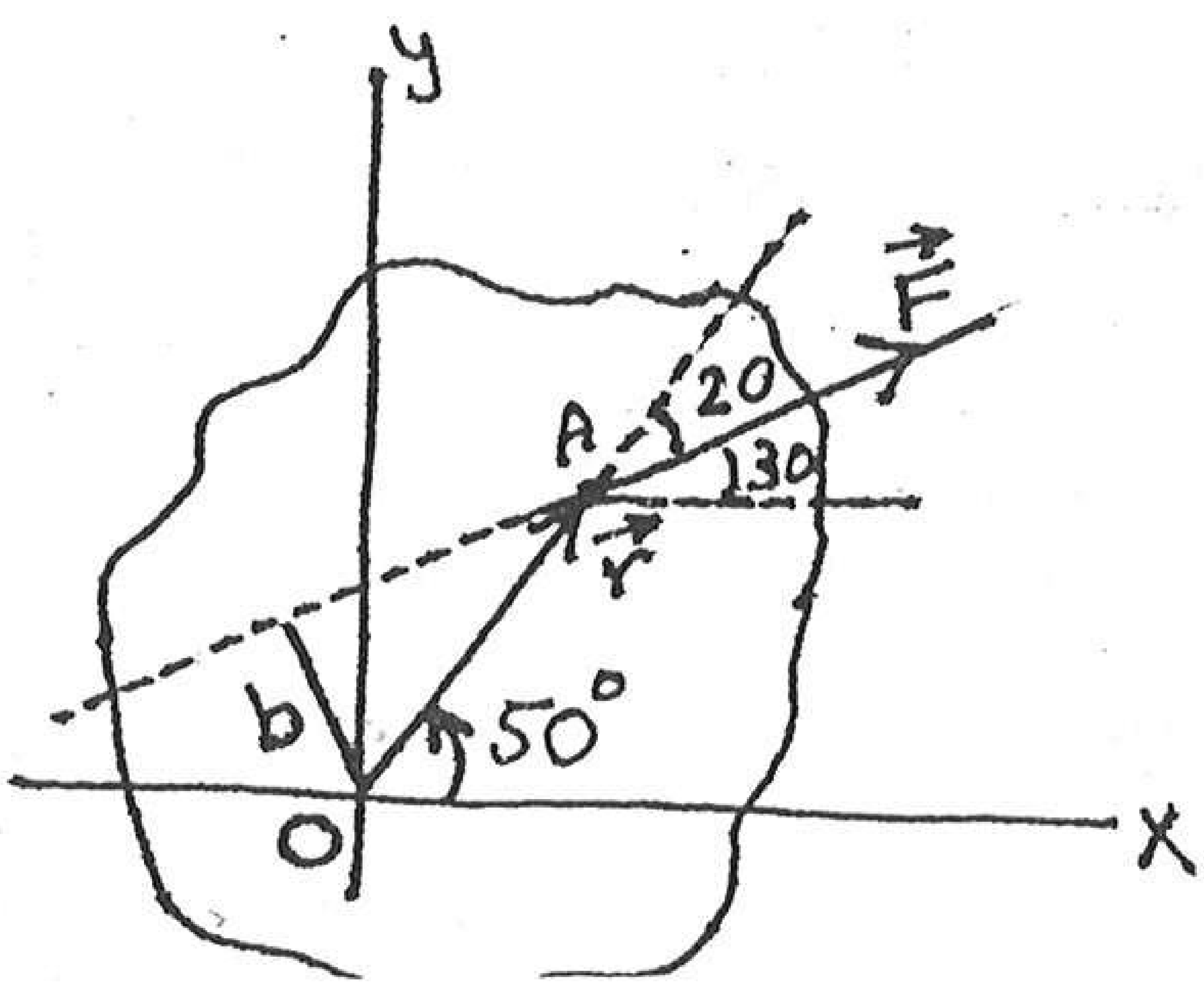
, $\vec{r}$: Position vector متجه الموضع

* In X-y plane: $\vec{r} = x\,i + y\,j$ and $\vec{F} = F_x\,i + F_y\,j$

$$\therefore \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ F_x & F_y & 0 \end{vmatrix} = K(xF_y - yF_x)$$

ex: Torque applied to the body in the fig., $F = 6N$, $r = 45\,cm$
Find The torque:

sol.: ① scalar Method:



$r = 45\,cm = 0.45\,m$

$b = r\sin 20^\circ = 0.45 * 0.342 = 0.154\,m$

$\therefore$ The torque around O is:

$\tau = Fb = (6)(0.154) = 0.924\ N \cdot m$

② Vector Method : $\vec{T} = \vec{r} \times \vec{F}$

$\vec{r} = xi + yj$, $\vec{F} = F_x i + F_y j$

$x = r\cos 50 = .45 \cos 50 = 0.289m$, $y = r\sin 50 = 0.345m$

$F_x = F\cos 30 = 6 \cos 30 = 5.196 N$, $F_y = F\sin 30 = 3 N$

$\therefore \vec{r} = 0.289 i + 0.345 j$, $\vec{F} = 5.196 i + 3j$

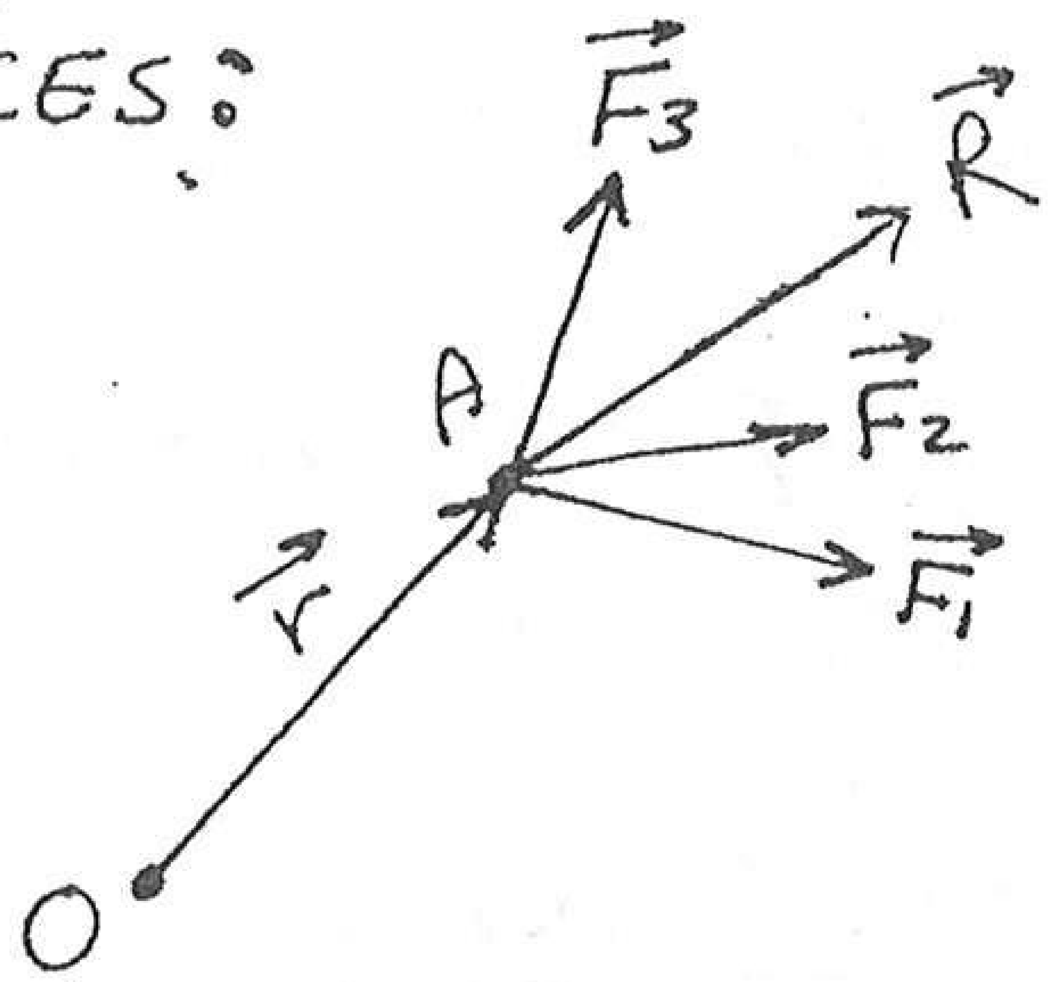
$$\vec{T} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & K \\ 0.289 & 0.345 & 0 \\ 5.196 & 3 & 0 \end{vmatrix} = (3)(0.289) - (5.196)(0.345)$$

الدوران باتجاه عقرب الساعة
يكون العزم اتجاه سالب

$$= 0.867 - 1.793 = K(-0.924) N.m$$

عزم عدة متلاقية قوى

* TORQUE OF SEVERAL CONCURRENT FORCES :



$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots$

$\vec{T} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F_1} + \vec{F_2} + \vec{F_3} + \dots)$

$\vec{T} = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3} + \dots$

$\therefore \vec{T} = \vec{T_1} + \vec{T_2} + \vec{T_3} + \dots = \sum T_i$

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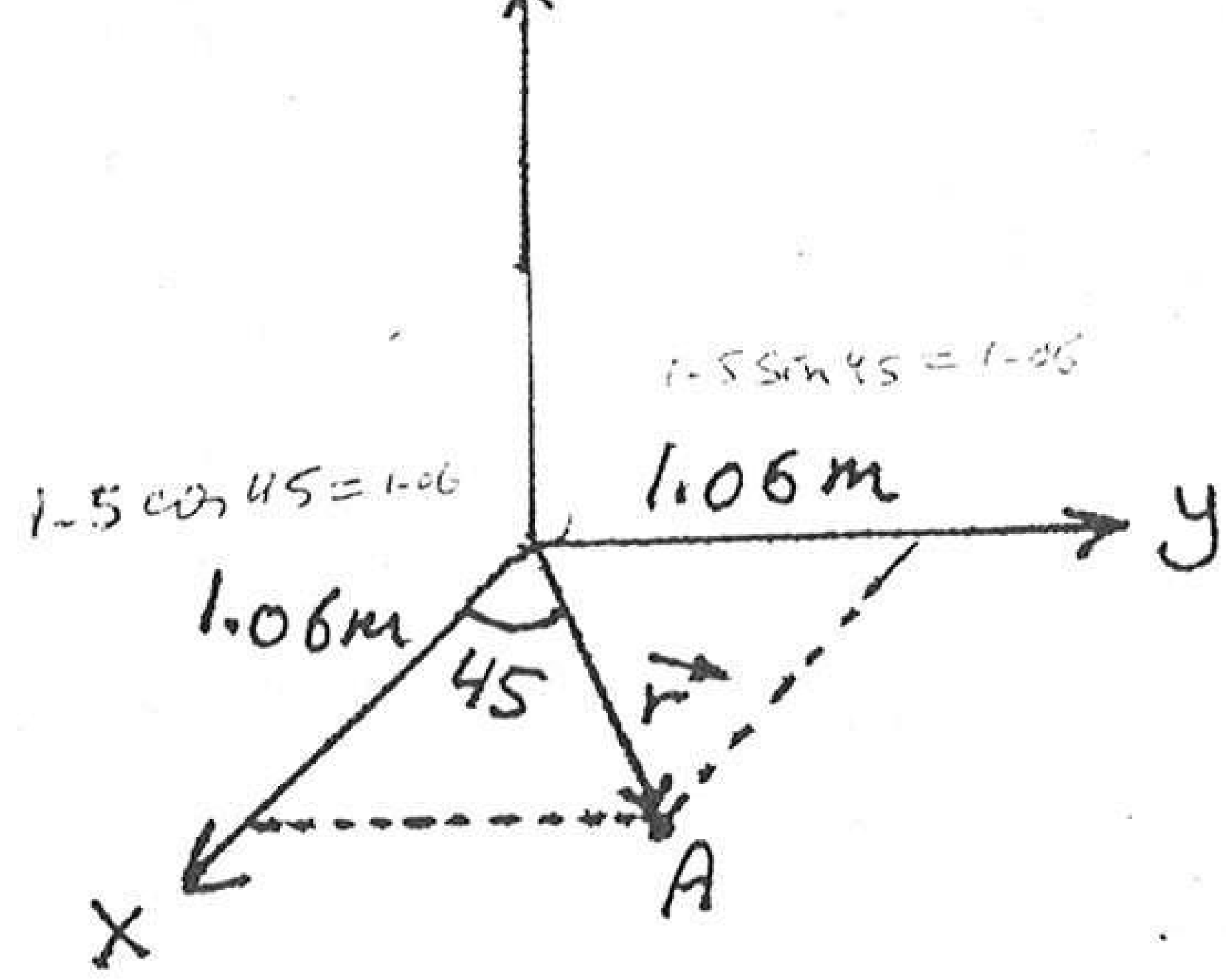
ex : Consider two forces applied at Point A in fig. where $r = 1.5m$ and $\vec{F_1} = 6i$, $\vec{F_2} = 6i - 7j$

Find the Resultant torque :



Sol. $\vec{T} = \vec{r} \times \vec{F} = \vec{r} \times \vec{R}$

$\vec{R} = \sum \vec{F_i} = \vec{F_1} + \vec{F_2} = 12i - 7j$

$\vec{r} = (1.06) i + (1.06) j$

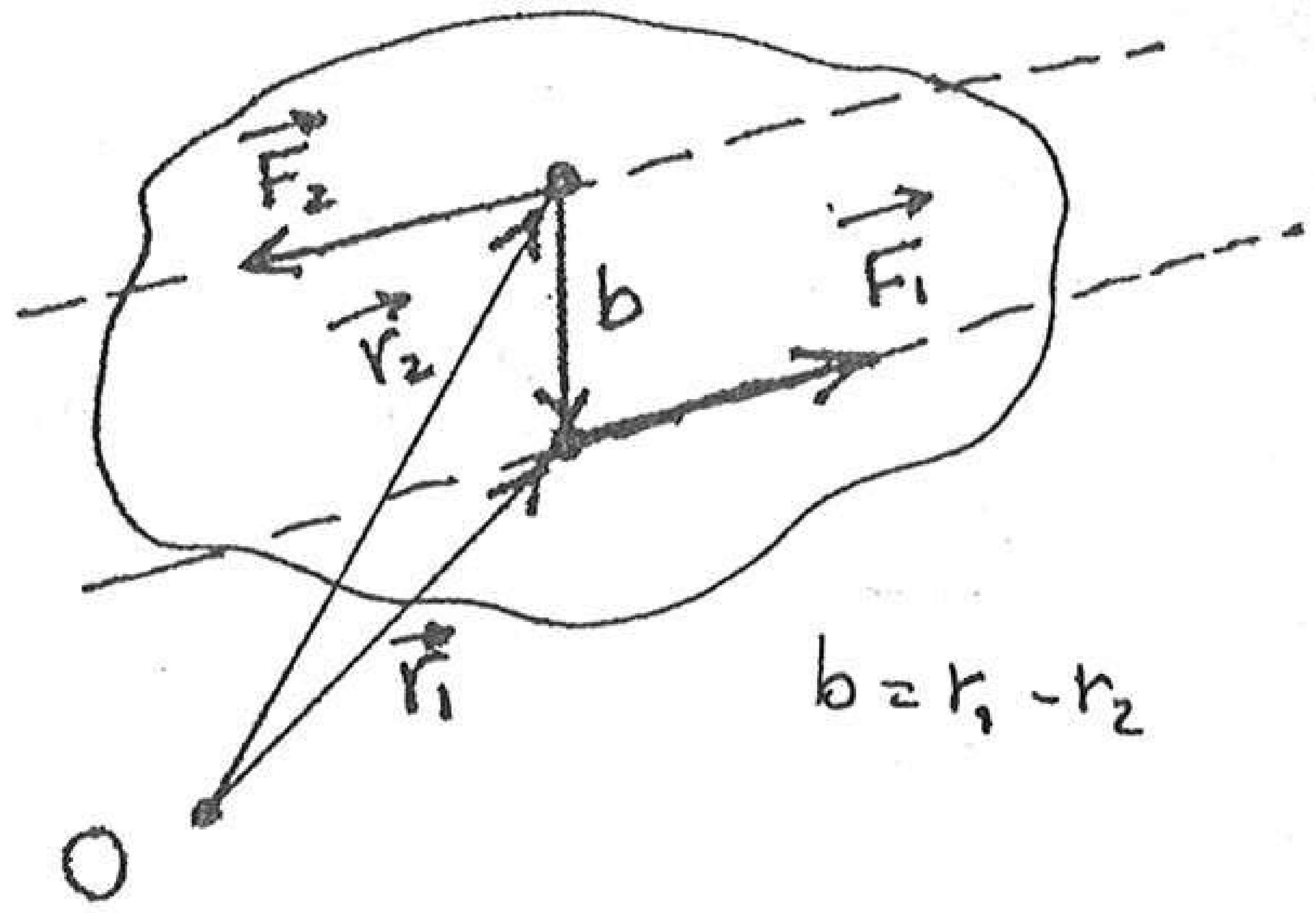
$$\therefore \vec{T} = \vec{r} \times \vec{R} = \begin{vmatrix} i & j & K \\ 1.06 & 1.06 & 0 \\ 12 & -7 & 0 \end{vmatrix}$$

$\vec{T} = -K(20.14)$

H.W : Solve above example if the forces are : $\vec{F_1} = -2i + j + 3K$, $\vec{F_2} = i - 2j + 3K$

Couple :

System of two forces of equal magnitude but opposite direction acting along parallel lines.



$$\vec{R} = \vec{F_1} + \vec{F_2} = 0 \Rightarrow \vec{F_2} = -\vec{F_1}$$

$$\vec{\tau} = \vec{\tau_1} + \vec{\tau_2} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} = \vec{r_1} \times \vec{F_1} - \vec{r_2} \times \vec{F_1}$$

$$\therefore \vec{\tau} = (\vec{r_1} - \vec{r_2}) \times \vec{F_1} \Rightarrow \boxed{\vec{\tau} = \vec{b} \times \vec{F_1}}$$

$\vec{b} = \vec{r_1} - \vec{r_2}$ lever arm.

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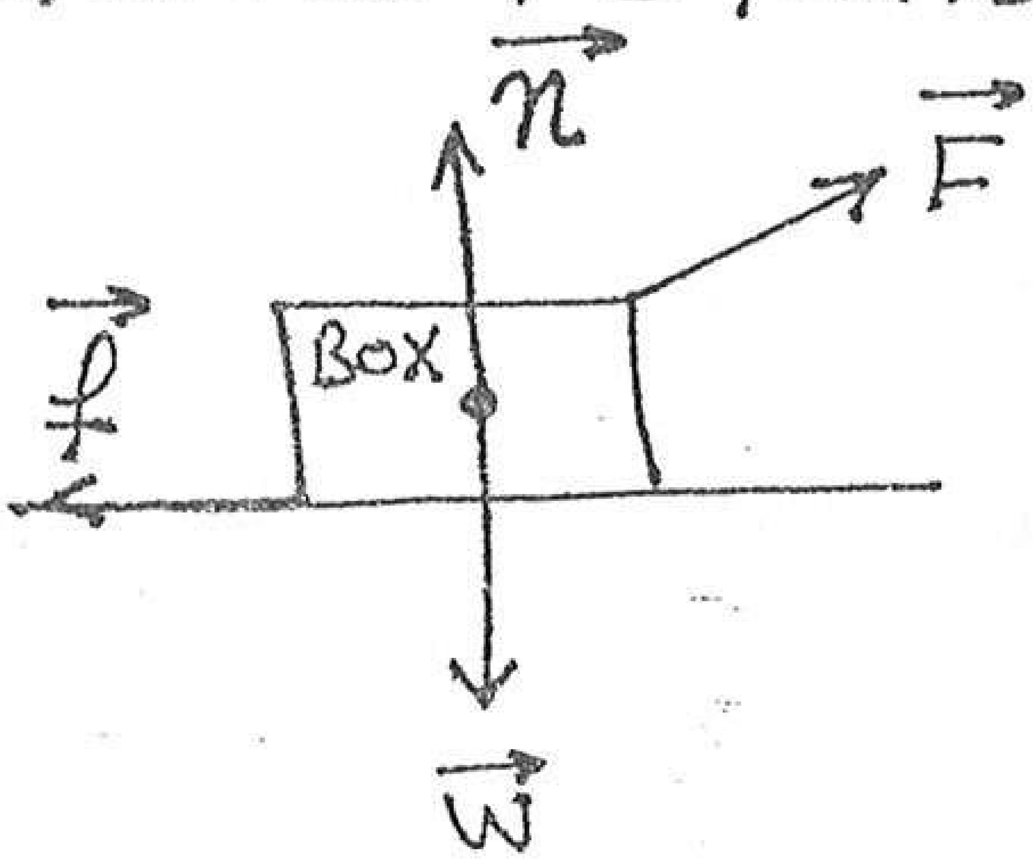
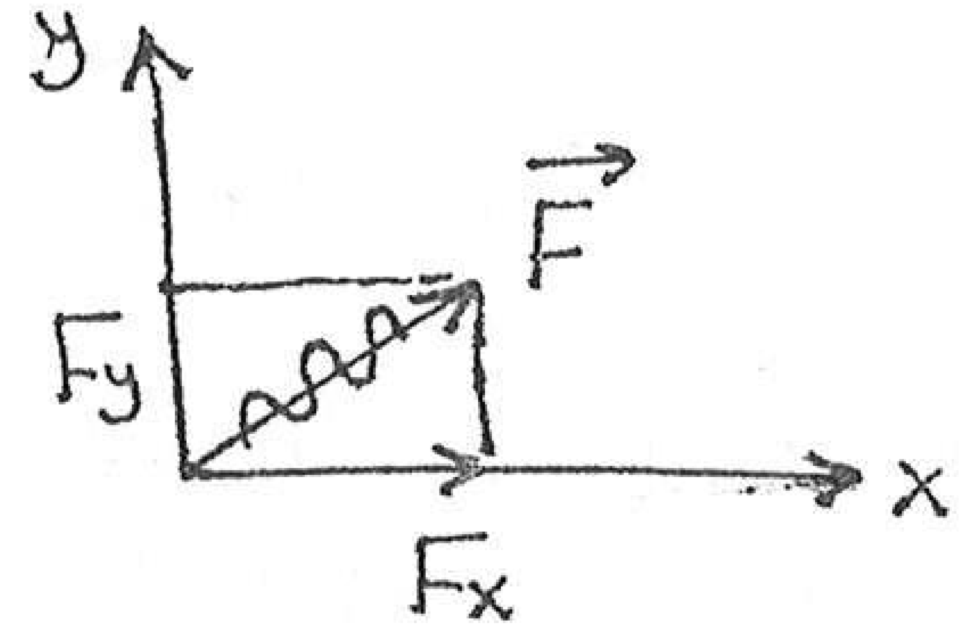
* Statics : Equilibrium of a Particle :



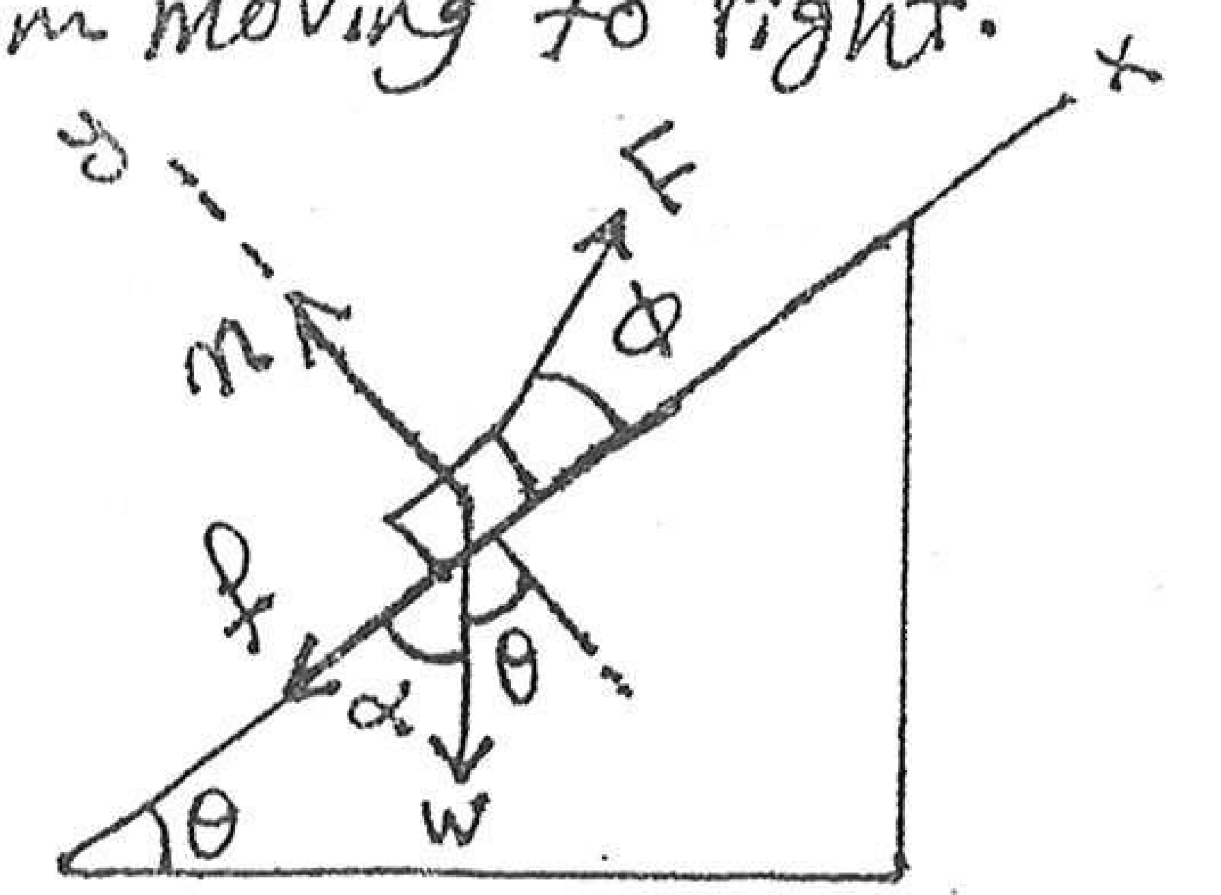


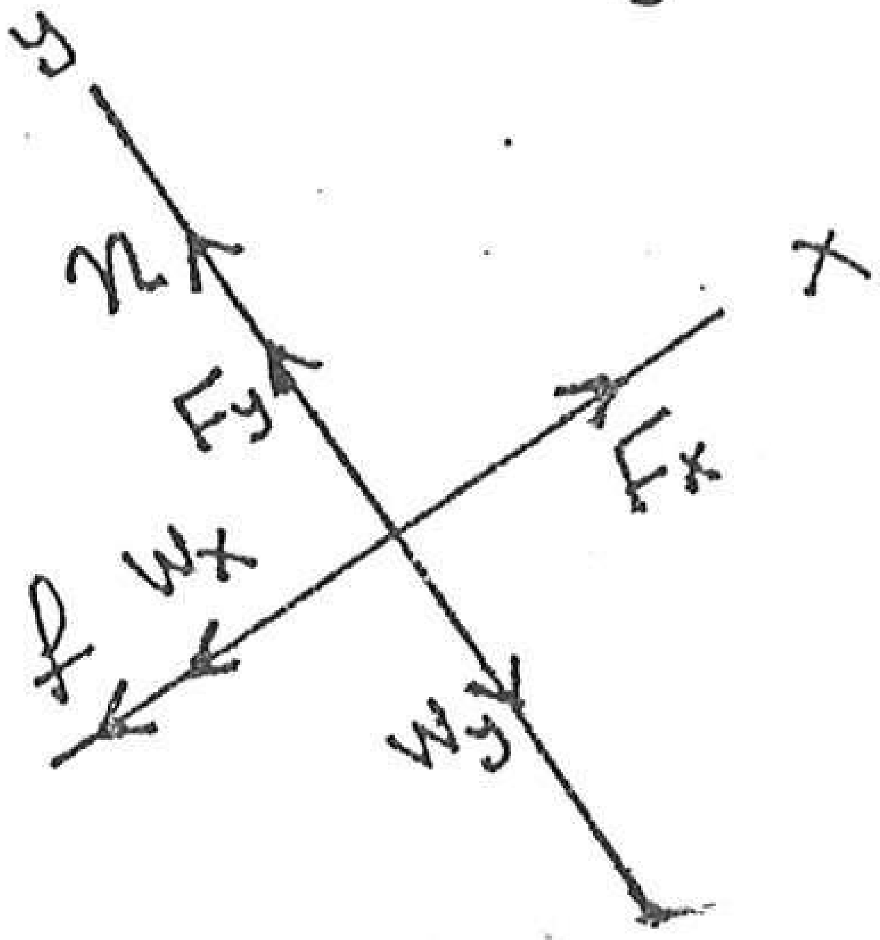
$\vec{F}$: force of man on box.

$\vec{n}$: force of contact of earth's surface against box.

$\vec{W}$: gravitational force of earth on box.

$\vec{f}$: friction force of earth against box tending to prevent box from moving to right.





$F_x = F\cos\phi$, $F_y = F\sin\phi$, $W_x = -W\sin\theta$, $W_y = -W\cos\theta$

$n_x = 0$, $n_y = n$, $f_x = -f$, $f_y = 0$

* Forces, if unbalanced, tend to change the state of motion of a body. A body is said to be in equilibrium if it:

① is at rest or moves at constant speed in a straight line.

② is either not rotating or is rotating at a constant rate.

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots = \sum \vec{F_i} = 0 \; ; \; \sum F_x = 0, \; \sum F_y = 0, \; \sum F_z = 0$$

These conditions are statements of Newton's First Law of motion

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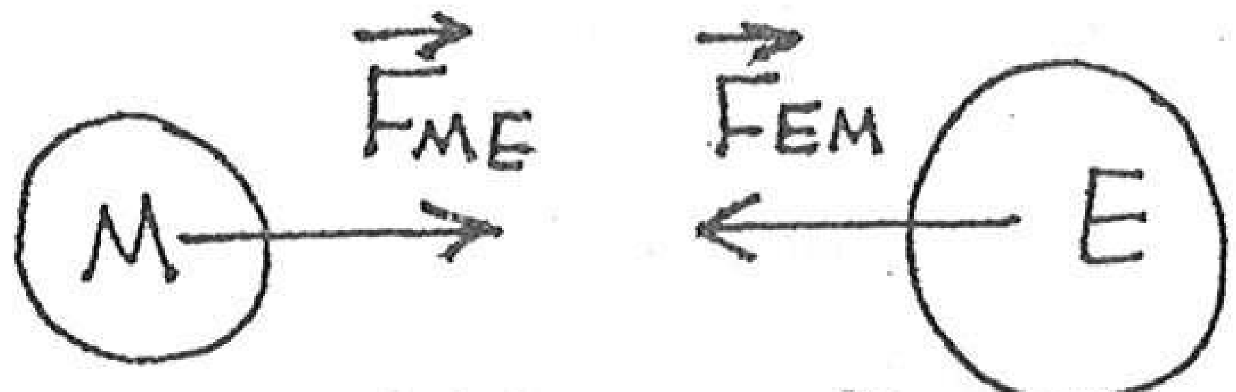


$$-\vec{F}_{ME} = \vec{F}_{EM} \quad \text{(action and reaction Pairs)}$$

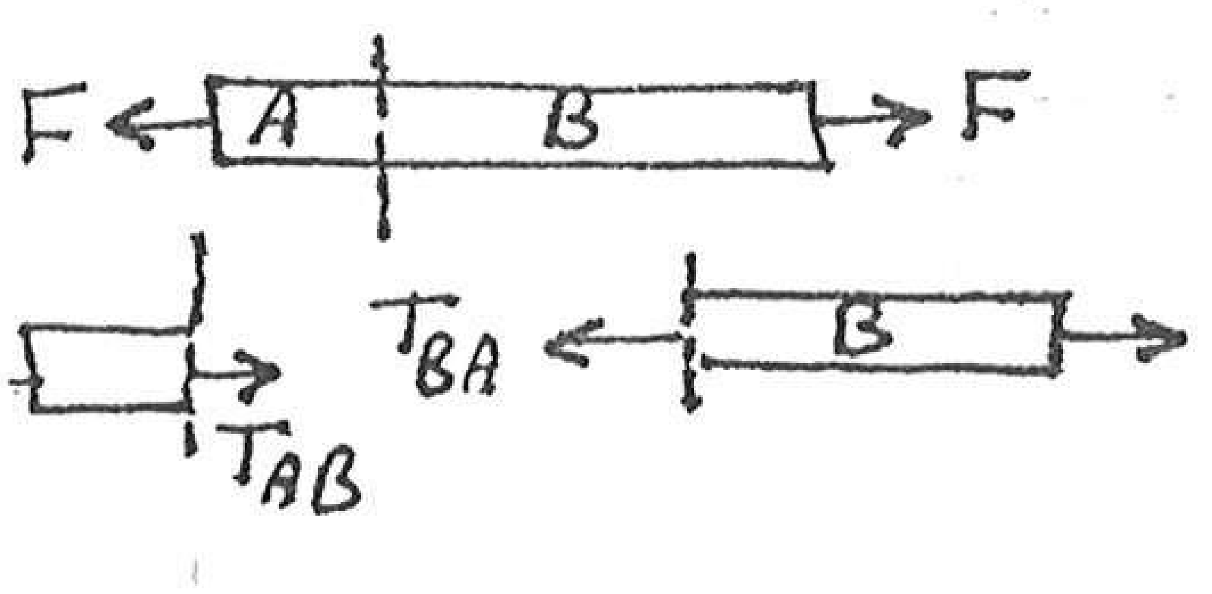
- Newton's third law of motion: when two Particles interact, the force on one Particle is equal and opposite to the force on the other.

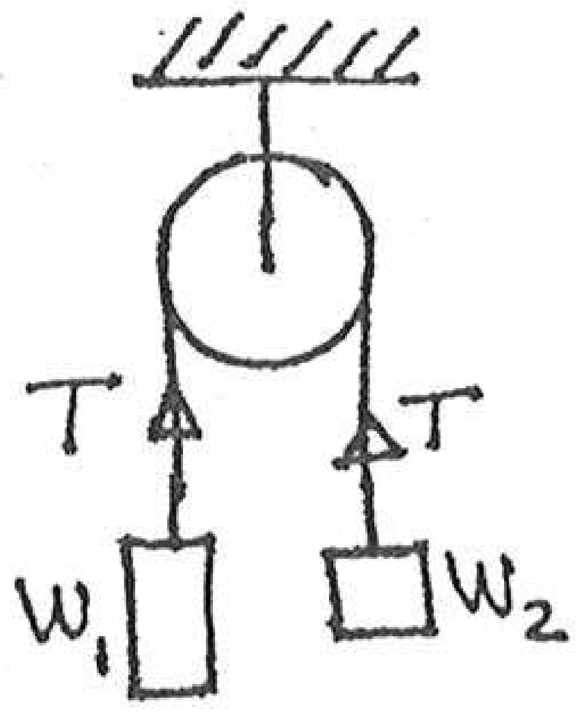
$\Rightarrow F' = -F$ force of box on man

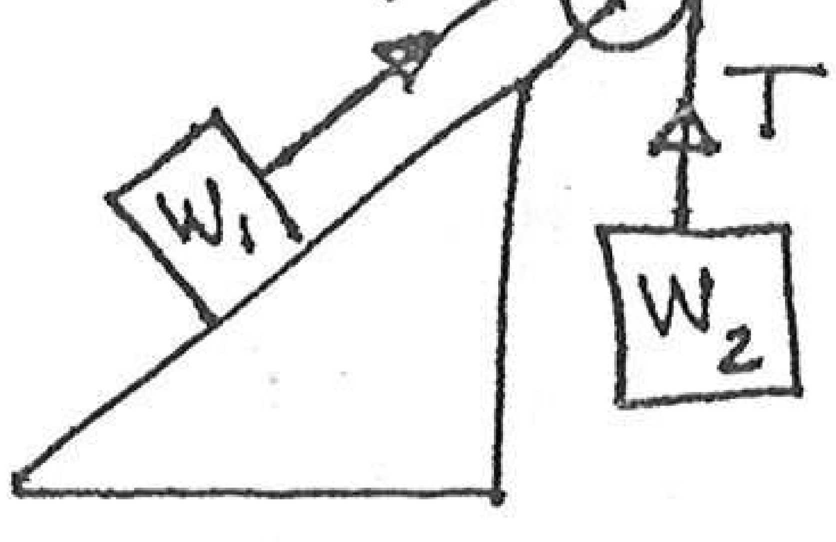
$n' = -n$ force of contact of box against earth's surface.

$W' = -W$ gravitational force of box on earth.

$f' = -f$ force of box on earth tending to drag earth to right.







Where: T is the tension force.

* **Friction**: The friction force f between two surfaces is parallel to the surfaces in a direction to oppose the relation motion which is occurring (يحدث) (Kinetic friction) (الاحتكاك الحركي).
or the motion which would occure if the friction were not there (static friction) (الاحتكاك السكوني).

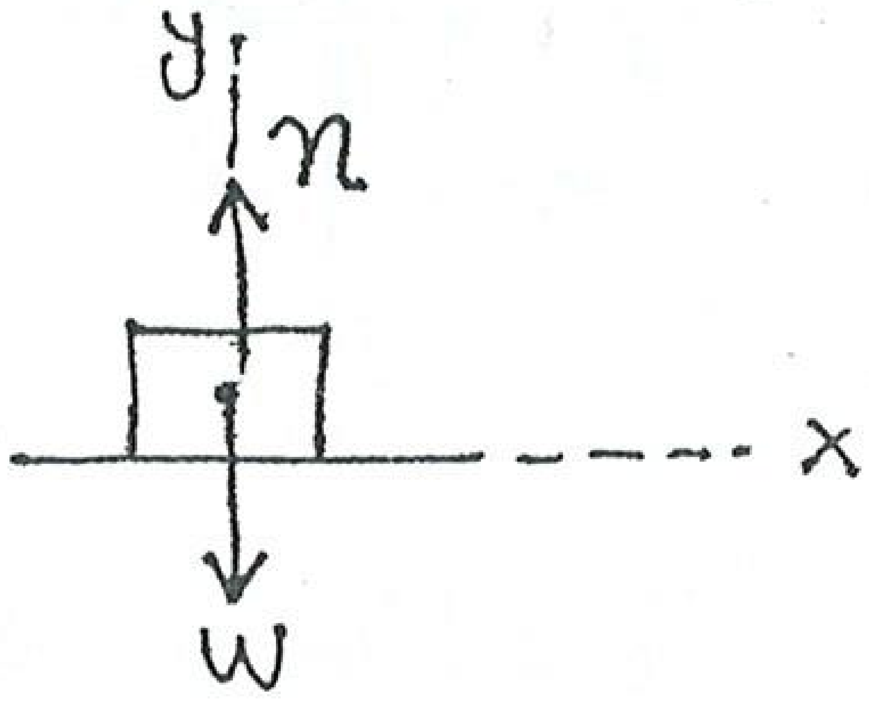
① static friction: $f_s \leq \mu_s n$ (μ_s is characteristic of the surfaces)

② Kinetic friction: $f_K = \mu_K n$ (when the external force, tending to move the surfaces over each other)

Note: $\mu_K < \mu_s$; $\mu < 1$ دائماً

علماً بأن μ_s أكبر من μ_K

ex1: Find all forces acting on the block in fig.; suppose a smooth (أملس) frictionless surface.



sol. ∵ The block in equilibrium; $\sum \vec{F} = 0$

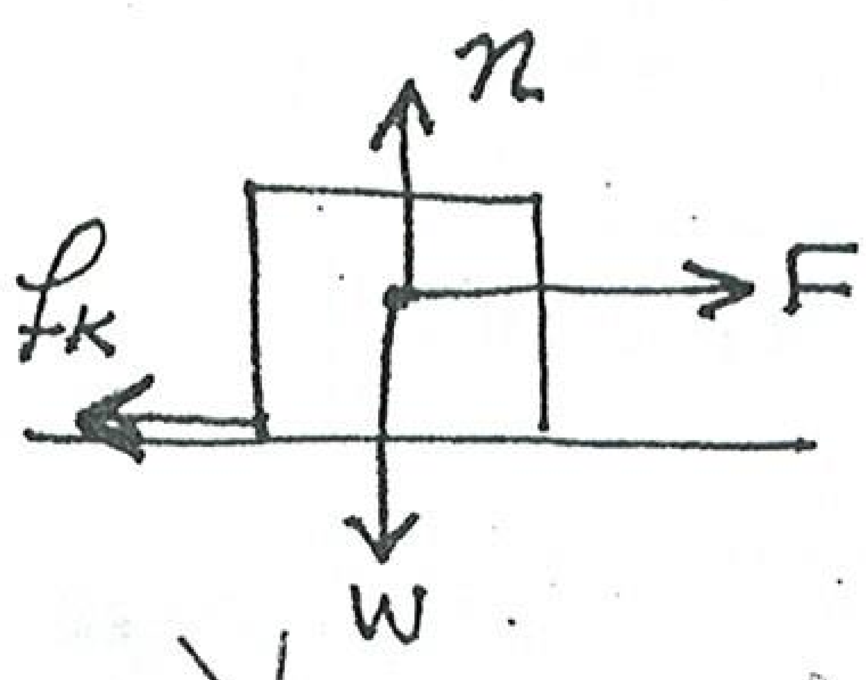
$\sum F_x = 0$

$\sum F_y = n - W = 0$

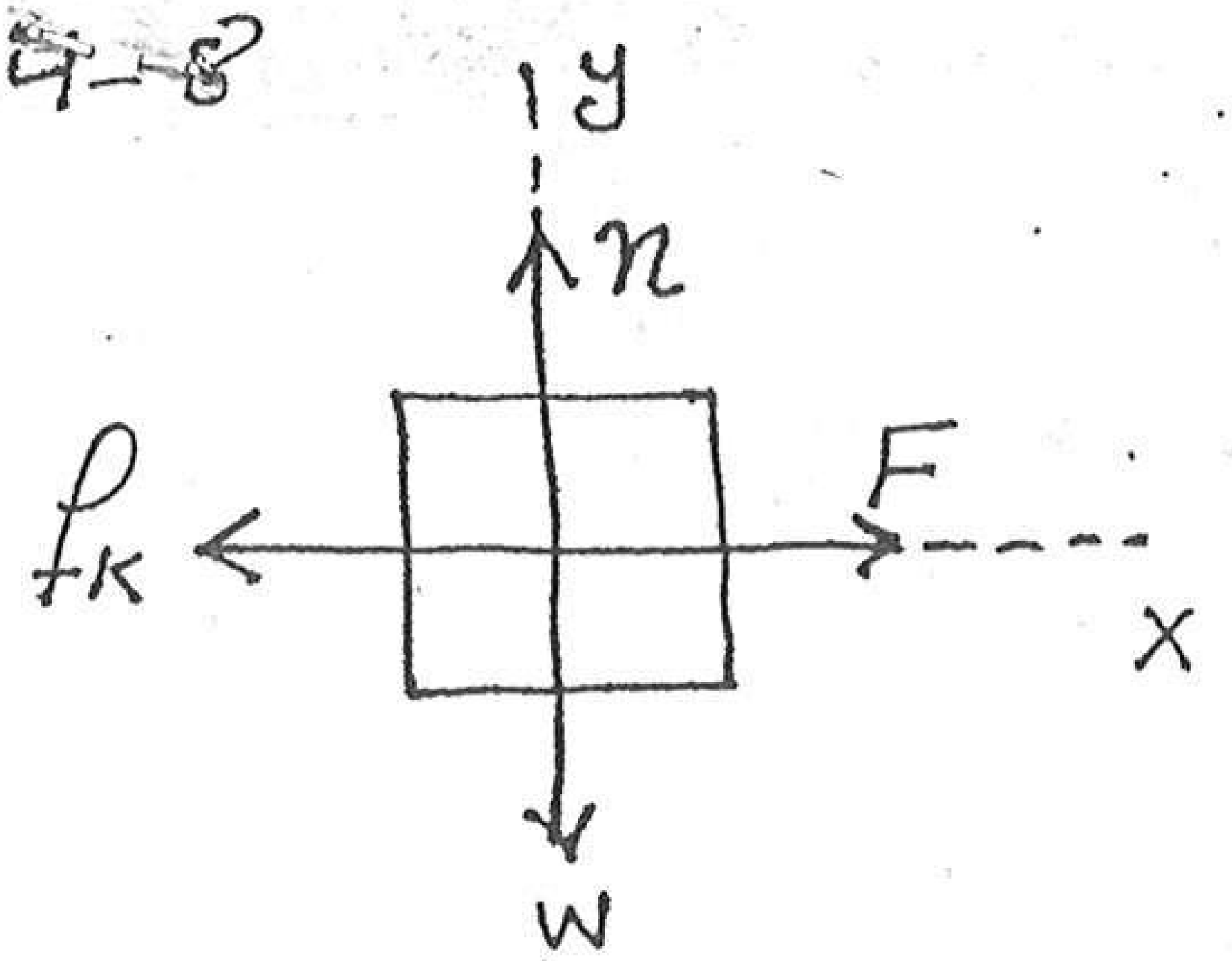
ex2: Consider the block pulled with constant velocity along a surface with friction by a force $\vec{F}$ ① find the force F in terms of μ_K and W.

② find μ_K if $W = 30N$ and $F = 10N$

③ find F if $\mu_K = 0.25$ and $W = 30N$



Sol.



"Free-body-Diagram"

① $\sum F = 0$

$\sum F_x = 0 = F - f_k$

$\Rightarrow F - \mu_k n = 0$

$\sum F_y = 0 = n - w \Rightarrow n = w$

$\therefore F = \mu_k n = \mu_k W$

② $\mu_k = \frac{F}{W} = \frac{10}{30} = \frac{1}{3}$

③ $F = \mu_k W = 0.25\,(30) = 7.5\,N$

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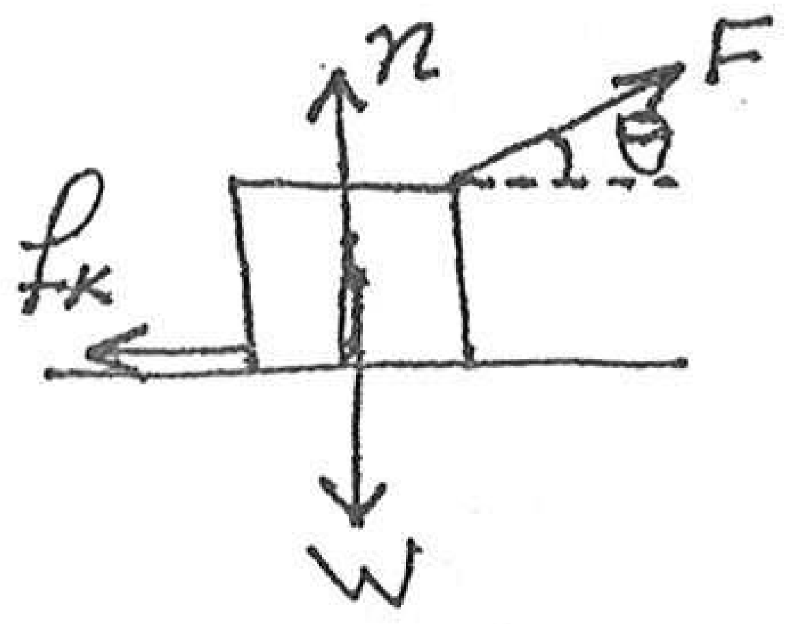
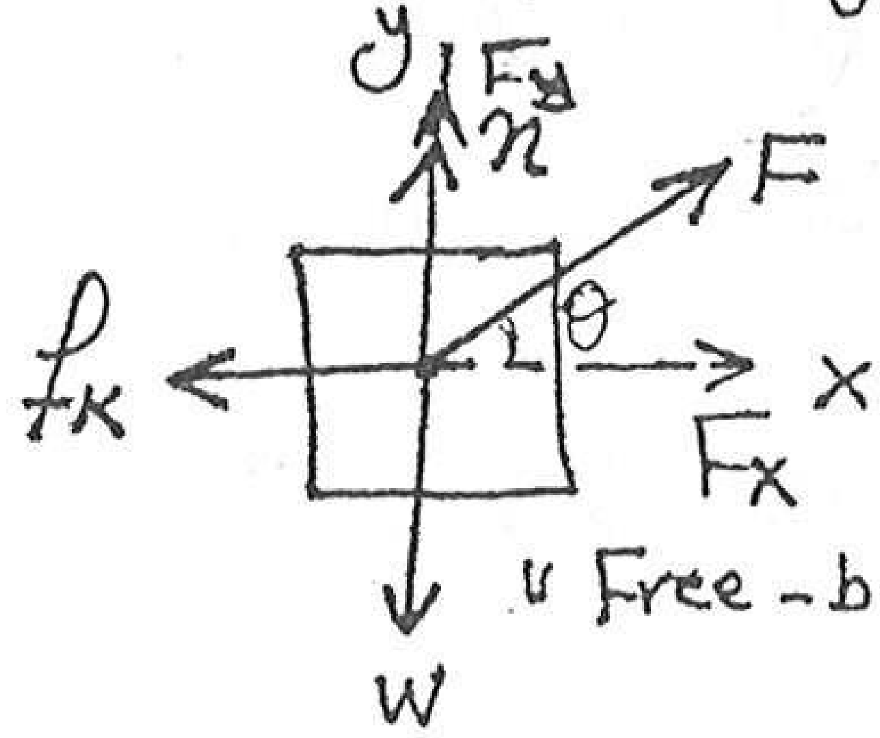
ex3: The block is moving uniformly against friction and the force F makes θ with x-axis, find F as a function of weight W and μ_k.

Sol.





"Free-body-Diagram"

from the conditions of equilibrium:

$\sum F_x = 0 = F\cos\theta - f_k = F\cos\theta - \mu_k n$ — ①

$\sum F_y = 0 = n + F\sin\theta - W$ — ②

from ② $n = W - F\sin\theta$ substitution in ①

$\therefore F\cos\theta - \mu_k (W - F\sin\theta) = 0$

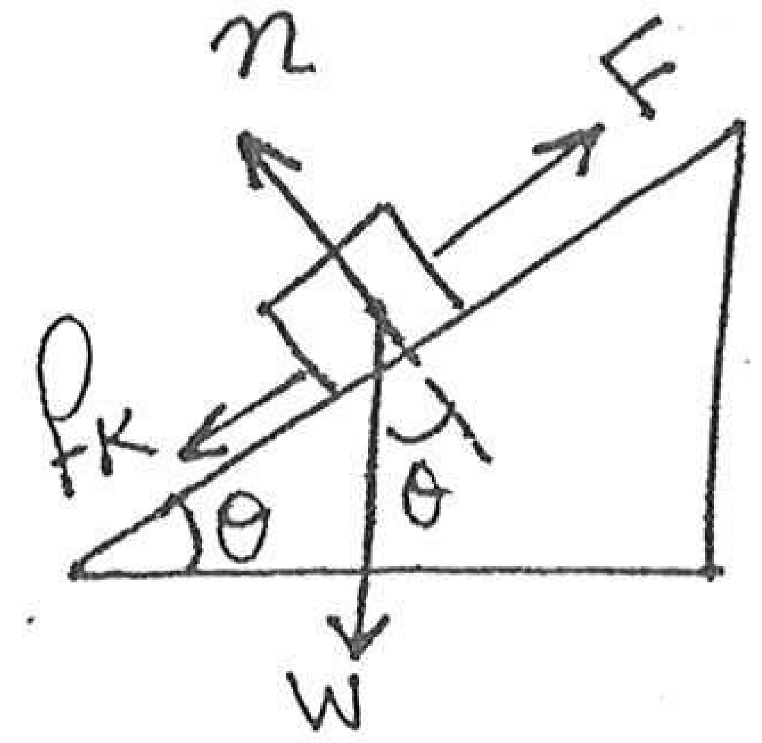
$\Rightarrow F(\cos\theta + \mu_k \sin\theta) - \mu_k W = 0$

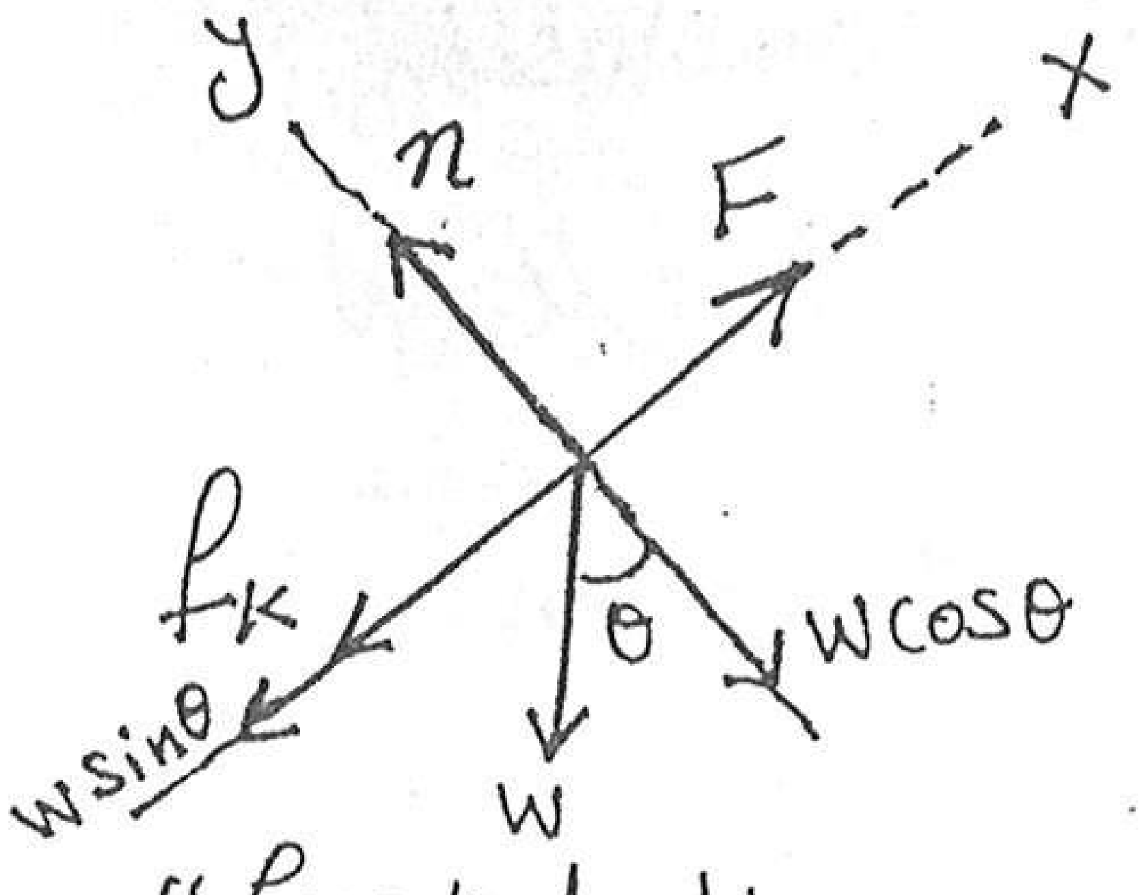
$$\Rightarrow \boxed{F = \mu_k \frac{W}{\cos\theta + \mu_k \sin\theta}}$$

Note: Calculate F if $\theta = 60°$ and $W = 20N$ and $\mu_k = 0.3$

ex4: A body of weight $W=10N$ moving uniformly up a plane inclined at $30°$ with coefficient of friction $\mu_k=0.3$, under the influence of an external force F pulling parallel to the plane, find the force F and n.

Sol.





" free body diagram "

$$\Sigma F_x = -f_k + F - W\sin\theta = 0$$

$$\Rightarrow -\mu_k n + F - W\sin\theta = 0 \quad \text{— (1)}$$

$$\Sigma F_y = n - W\cos\theta = 0 \Rightarrow \boxed{n = W\cos\theta} \quad \text{— (2)} \quad n = 10\cos 30 = 8.6N$$

substitute eq. (2) in (1)

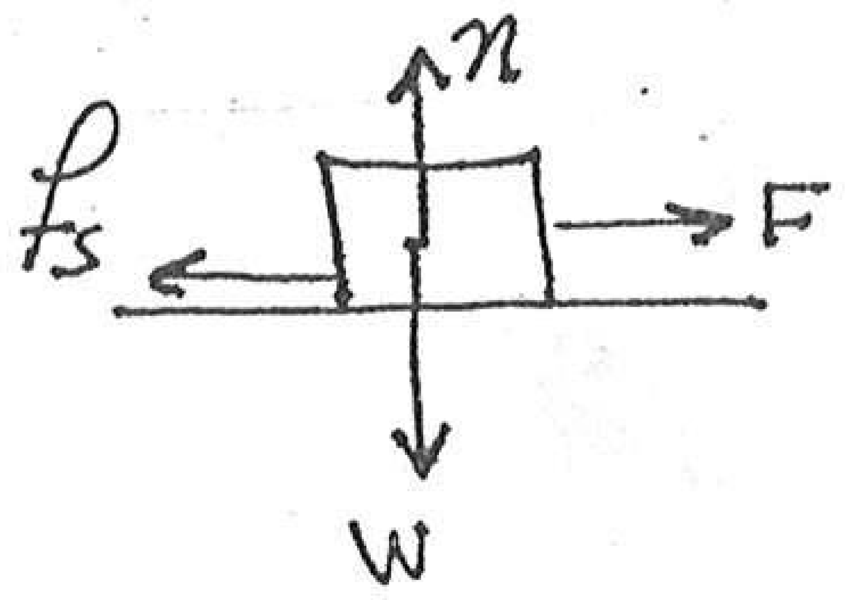
$$\therefore -\mu_k W\cos\theta + F - W\sin\theta = 0$$

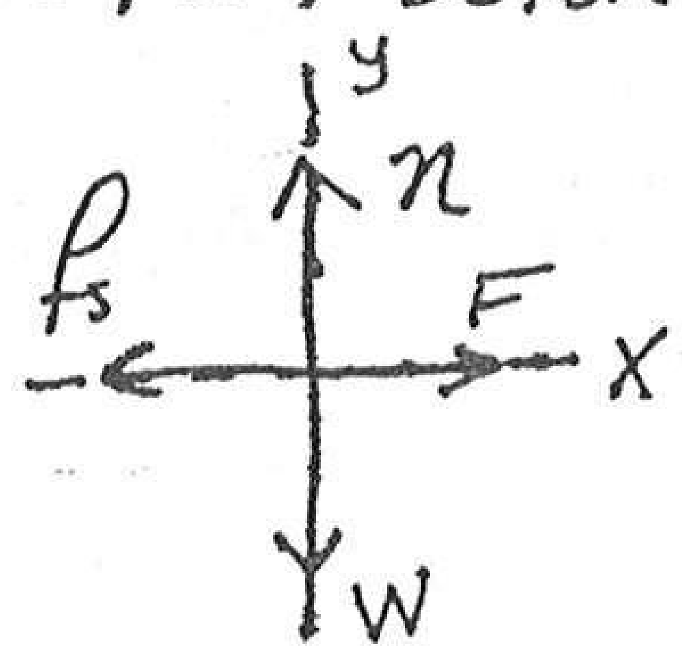
or $F = W(\sin\theta + \mu_k \cos\theta)$

$$\Rightarrow F = (10)(\sin 30 + 0.3\cos 30) = 7.6N.$$

ex5: A block of weight W is at rest on a rough surface of coefficient of static friction μ_s under the influence of horizontal force F. Find the range of values possible for F before the block moves.

Sol.





$\Sigma F_x = F - f_s = 0$ but $f_s \leq \mu_s n$

$\Sigma F_y = n - W = 0$

Note the incorrect conclusion $f_s = \mu_s n$. The static friction f_s depends on F. If $F=0$ the block just sits there and $f_s = 0$. The correct conclusion is $f_s = F$ when:

$f_s \leq \mu_s n = \mu_s W$ or $F \leq \mu_s W$

أخطأنا

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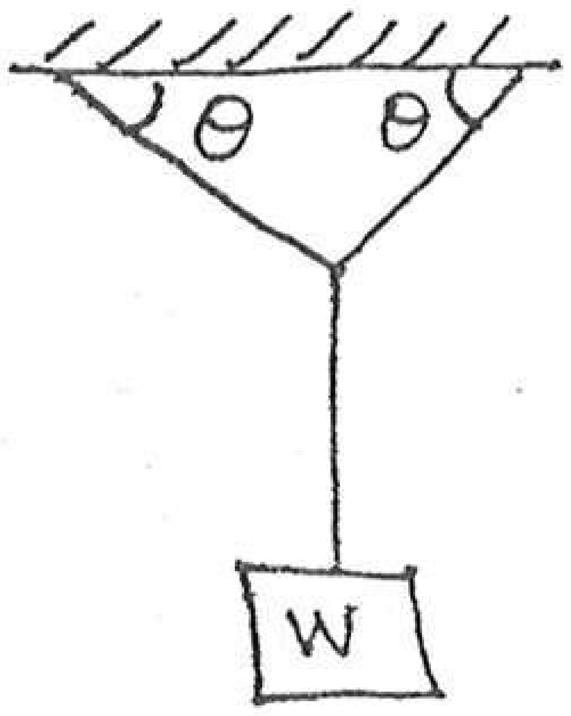
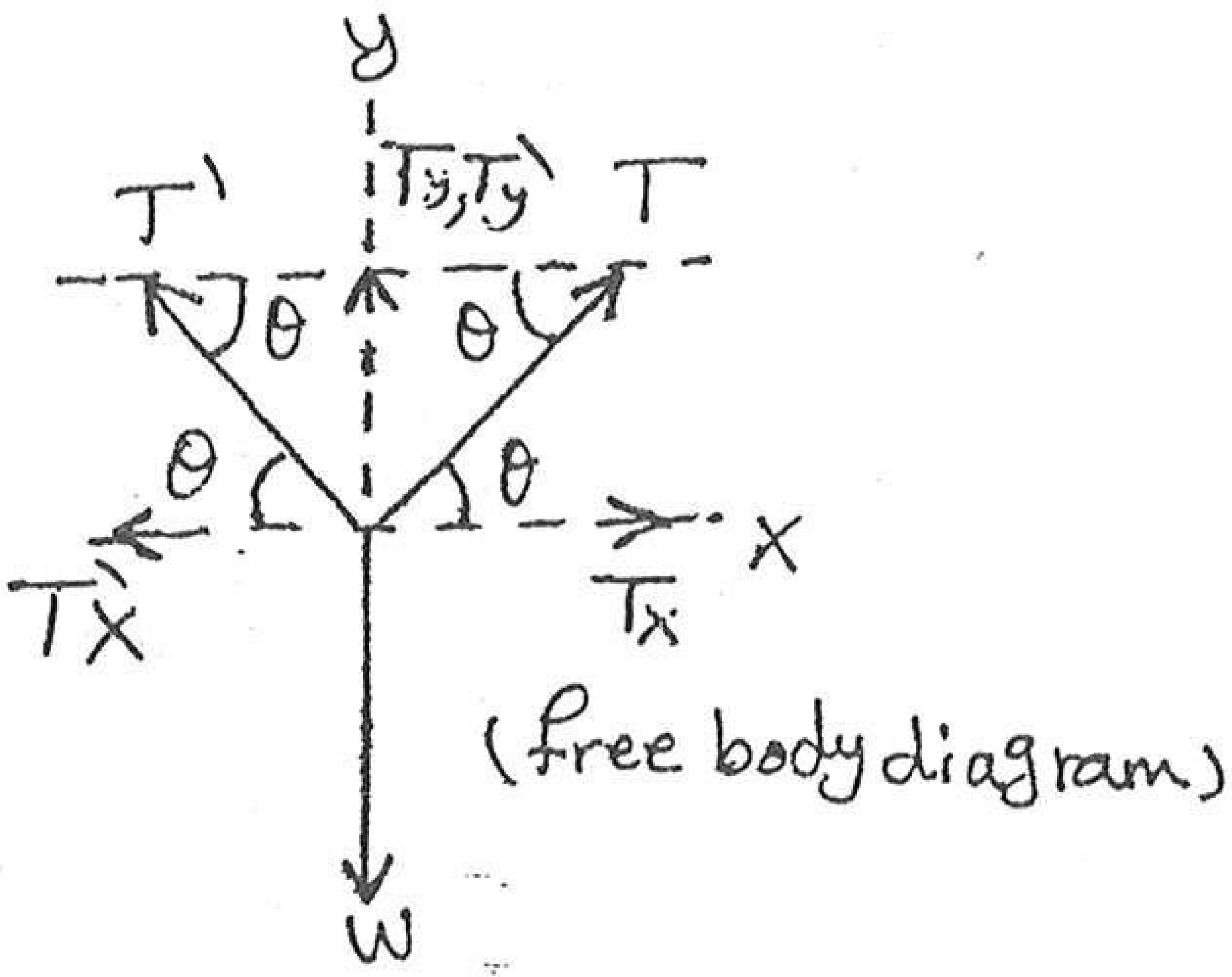
ex6: Find the tension in each string in fig:



Sol.



(free body diagram)

$\Sigma F_x = T\cos\theta - T'\cos\theta = 0 \Rightarrow T = T'$ — (1)

$\Sigma F_y = T\sin\theta + T'\sin\theta - W = 0$ — (2)

$\therefore T\sin\theta + T\sin\theta - W = 0 \Rightarrow 2T\sin\theta = W$

$\Rightarrow \boxed{T = \dfrac{W}{2\sin\theta}}$

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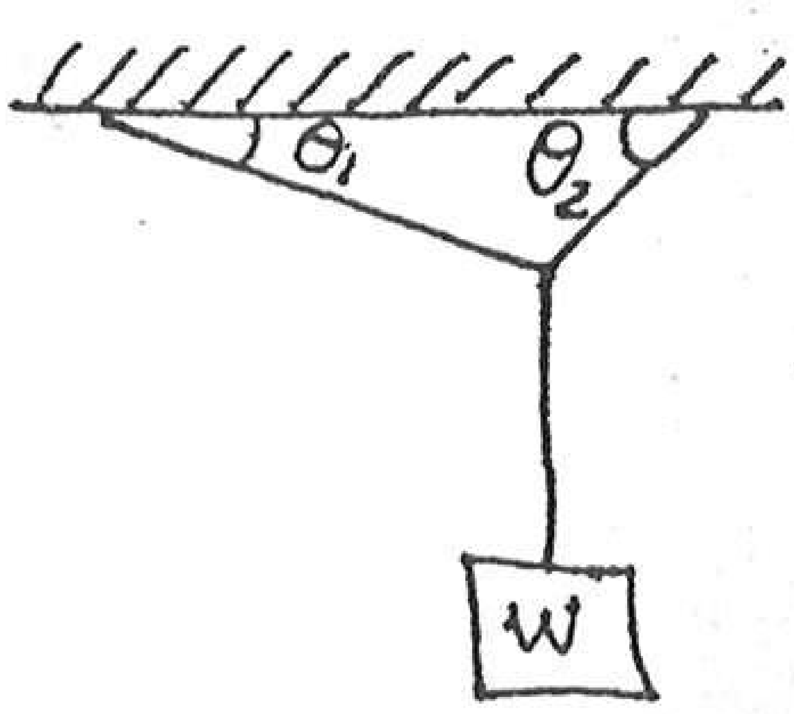
ex7: Find the tension in each string in fig:



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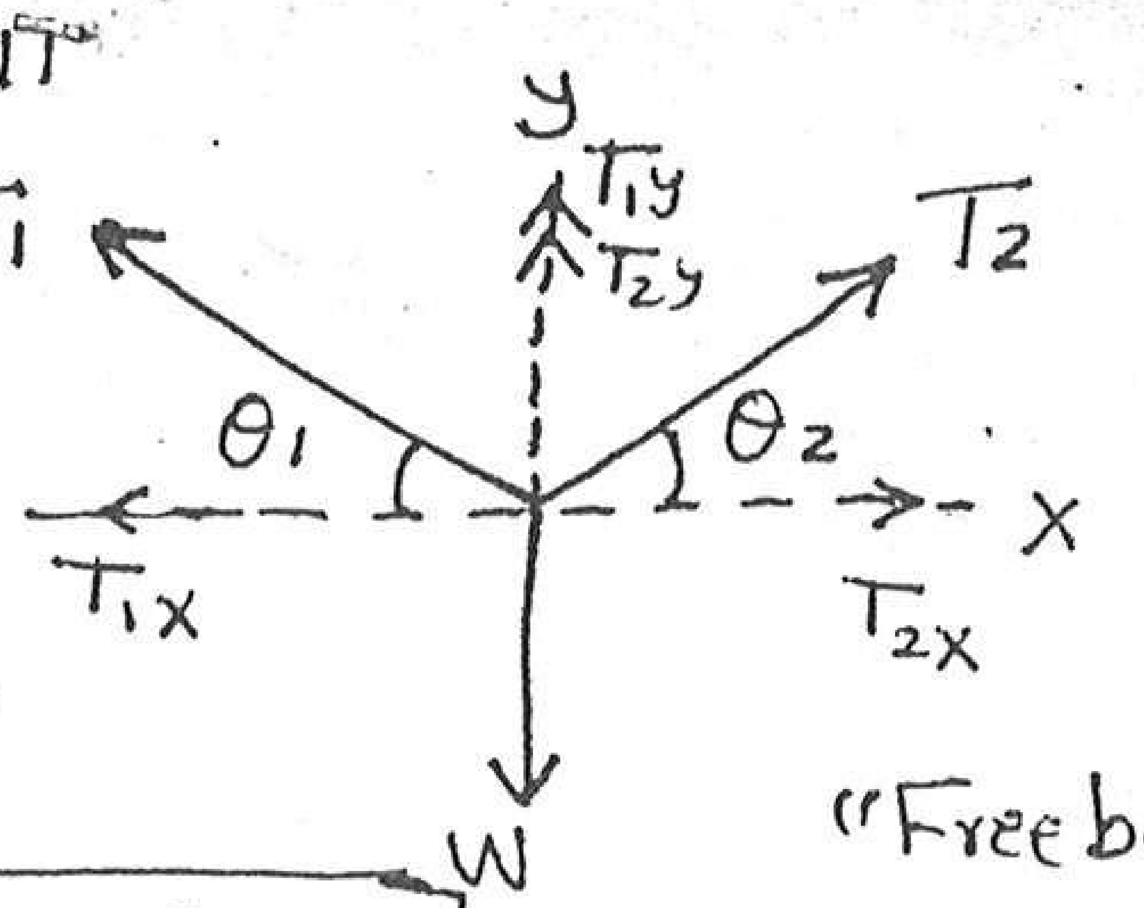
<u>sol.</u>



① $\Sigma F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$

② $\Sigma F_y = T_2 \sin\theta_2 + T_1 \sin\theta_1 - W = 0$

from ① $\dfrac{T_2}{T_1} = \dfrac{\cos\theta_1}{\cos\theta_2}$ or $\boxed{T_2 = T_1 \dfrac{\cos\theta_1}{\cos\theta_2}}$

$\Rightarrow$ from ② $T_1 \dfrac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 + T_1 \sin\theta_1 = W$

$$\Rightarrow \boxed{T_1 = \frac{W}{\sin\theta_1 + \left(\frac{\cos\theta_1}{\cos\theta_2}\right)\sin\theta_2}}$$

$$\text{or } T_1 = \frac{W}{\frac{\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2}{\cos\theta_2}} = \frac{W\cos\theta_2}{\sin(\theta_1+\theta_2)}$$ تبسيط

<u>also</u>: ① If $\theta_1 = \theta_2 = \theta \Rightarrow T_1 = T_2 = \dfrac{W\cos\theta}{\sin 2\theta} = \dfrac{W\cancel{\cos\theta}}{2\sin\theta\cancel{\cos\theta}} = \dfrac{W}{2\sin\theta}$

② If $\theta_1 + \theta_2 = 90^\circ$ then $T_1 = \dfrac{W\cos\theta_2}{\sin 90} = W\cos\theta_2 = W\sin\theta_1$

and $T_2 = T_1 \dfrac{\cos\theta_1}{\cos\theta_2}$

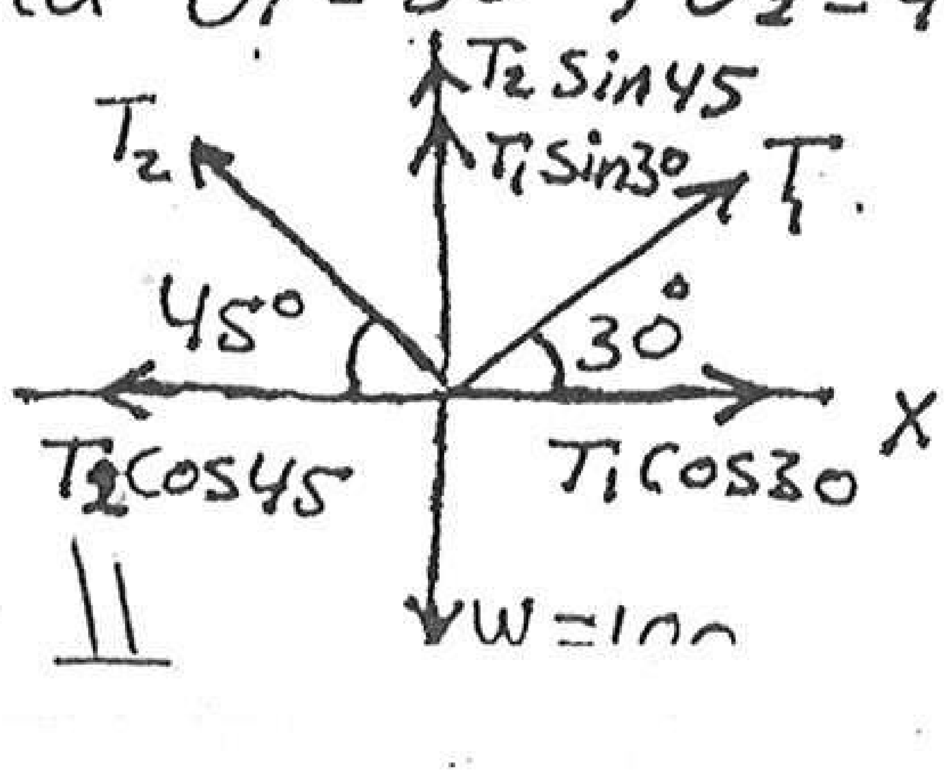
either $T_2 = W\cancel{\cos\theta_2}\dfrac{\cos\theta_1}{\cancel{\cos\theta_2}} = W\cos\theta_1$

or $T_2 = W\sin\theta_1 \dfrac{\cos\theta_1}{\cos\theta_2} = W\underbrace{\sin(90-\theta_2)}_{\cos\theta_2} \dfrac{\overbrace{\cos(90-\theta_2)}^{\sin\theta_2}}{\cos\theta_2}$

$T_2 = W\cancel{\cos\theta_2}\dfrac{\sin\theta_2}{\cancel{\cos\theta_2}} = W\sin\theta_2$

<u>H.W</u>: find T_1 and T_2 if $W = 100\,N$ and $\theta_1 = 30^\circ$, $\theta_2 = 45^\circ$

<u>Note</u>: use $\Sigma F_x = 0$ and $\Sigma F_y = 0$



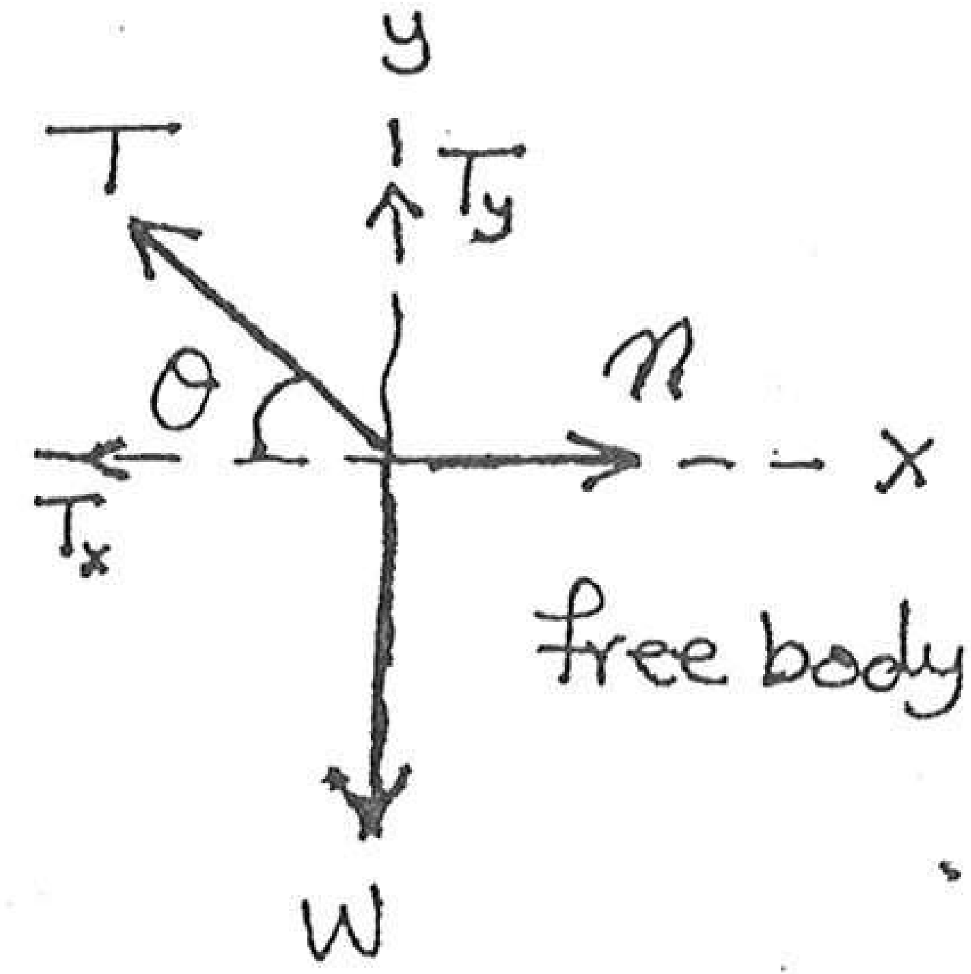
$\Sigma F_x = 0 \Rightarrow T_1 \cos 30 - T_2 \cos 45 = 0$ —①

$\Sigma F_y = 0 \Rightarrow T_2 \sin 45 + T_1 \sin 30 - 100 = 0$ —②

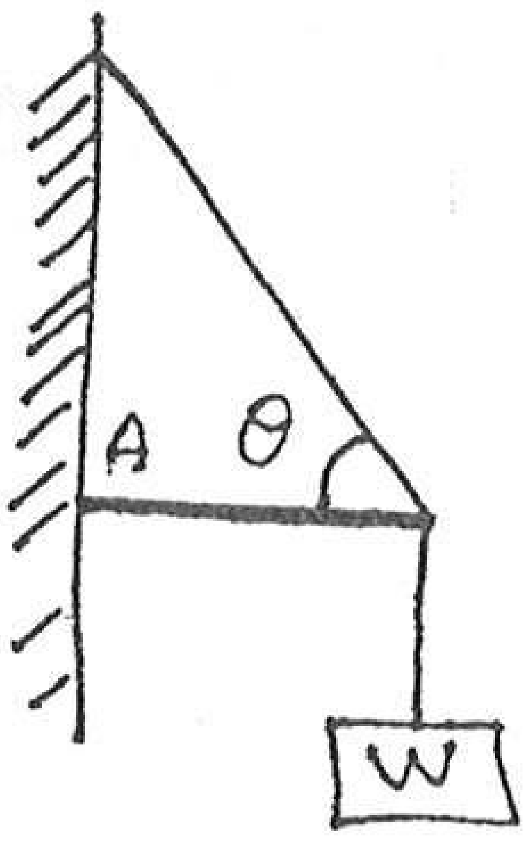
Quiz in these three examples

ex 8: Consider the weightless boom and cable configuration in fig. suppose that the only force of the wall on the boom is a perpendicular force n, as if the boom and wall were unattached and perfectly frictionless. The system is in equilibrium. Find the tension T and normal force n if $\theta = 60^\circ$ and $w = 100\,N$.

Sol.



free body diagram



$$\Sigma F_x = 0 \Rightarrow n - T\cos\theta = 0 \Rightarrow n = T\cos\theta \quad —— (1)$$

$$\Sigma F_y = 0 \Rightarrow T\sin\theta - w = 0 \Rightarrow T = \frac{w}{\sin\theta} \quad —— (2)$$

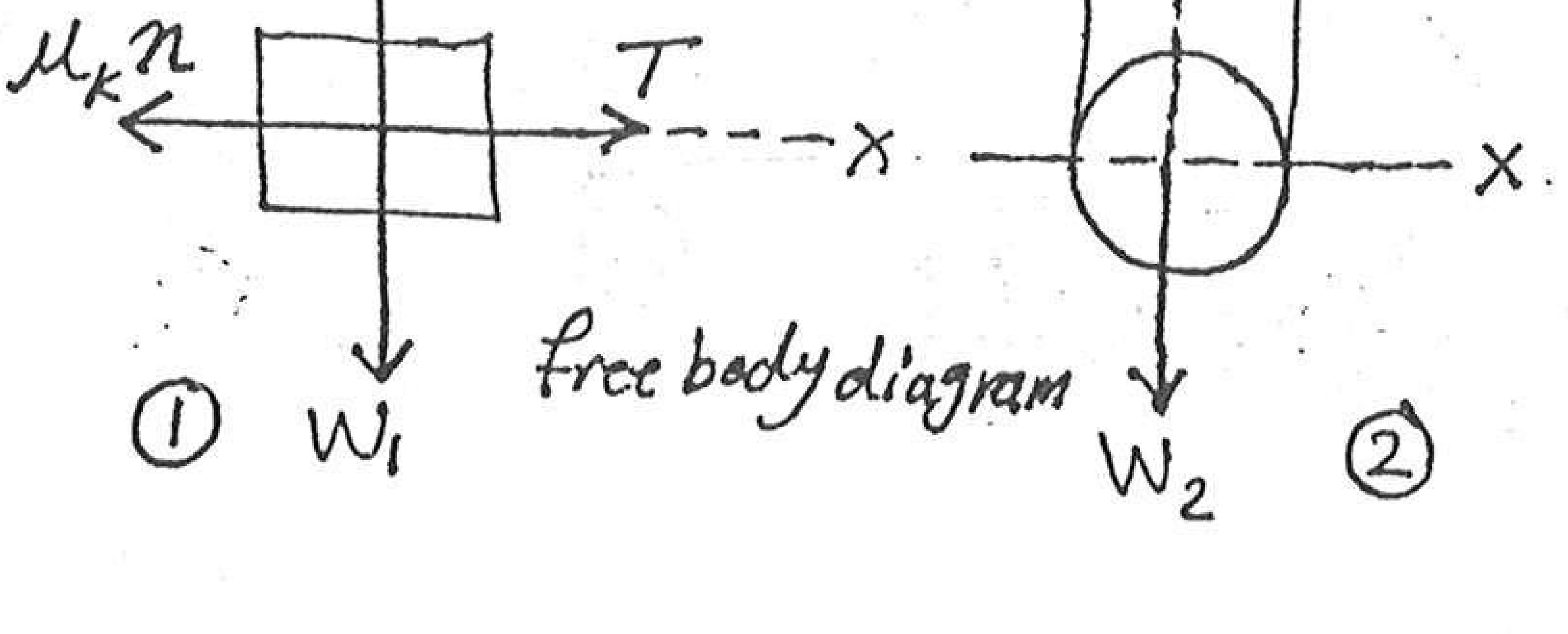
$$\therefore n = \frac{w}{\sin\theta}\cos\theta = \frac{w}{\tan\theta} = \frac{100}{\tan 60} = 58\,N$$

$$T = \frac{w}{\sin\theta} = \frac{100}{0.866} = 115\,N$$

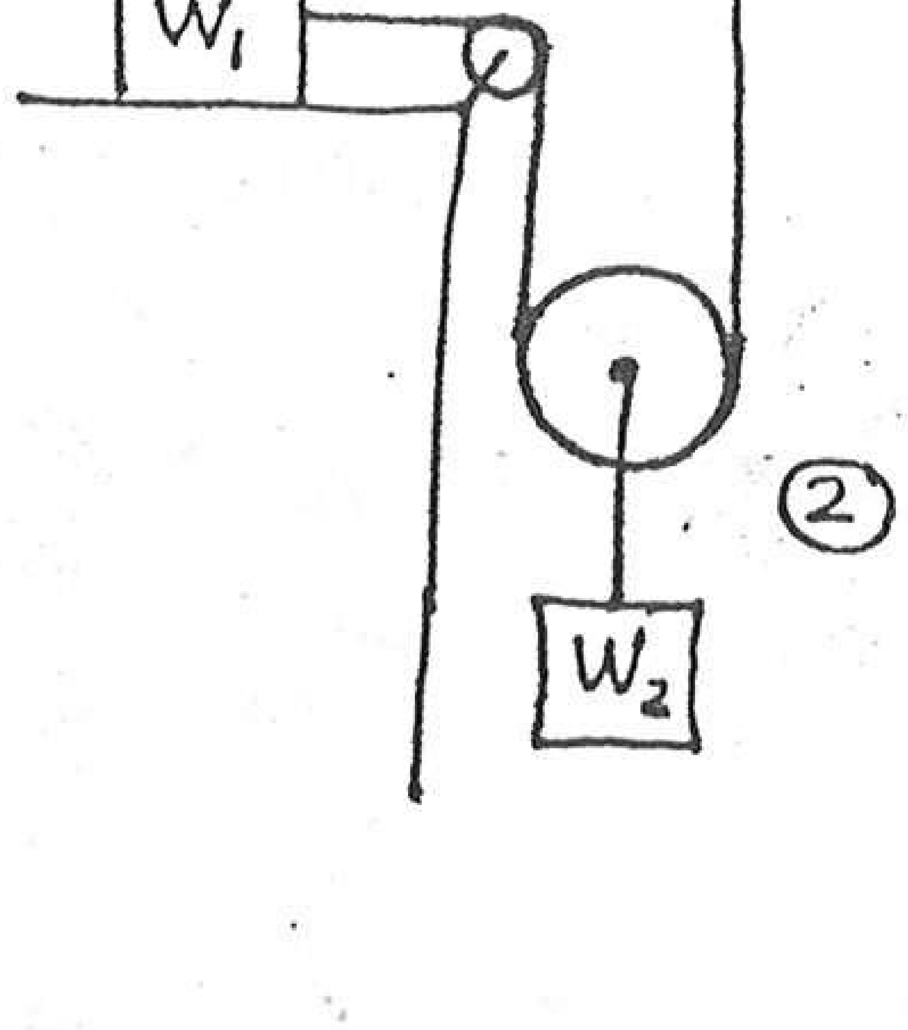
---

ex 9: Calculate the cofficient of kinetic friction between the block W_1 and the surface. If $W_1 = 5\,N$ moves at constant speed and $W_2 = 2\,N$.

Sol.



free body diagram



- For W_1 the equilibrium conditions are:

$$\sum F_x = T - \mu_k' n = 0$$

$$\sum F_y = n - W_1 = 0 \Rightarrow n = W_1$$

$$\therefore \boxed{T = \mu_k W_1} \quad \text{—— (1)}$$

- For W_2 we have:

$\sum F_x = 0$,

$$\sum F_y = 2T - W_2 = 0 \Rightarrow \boxed{T = \frac{W_2}{2}} \quad \text{—— (2)}$$

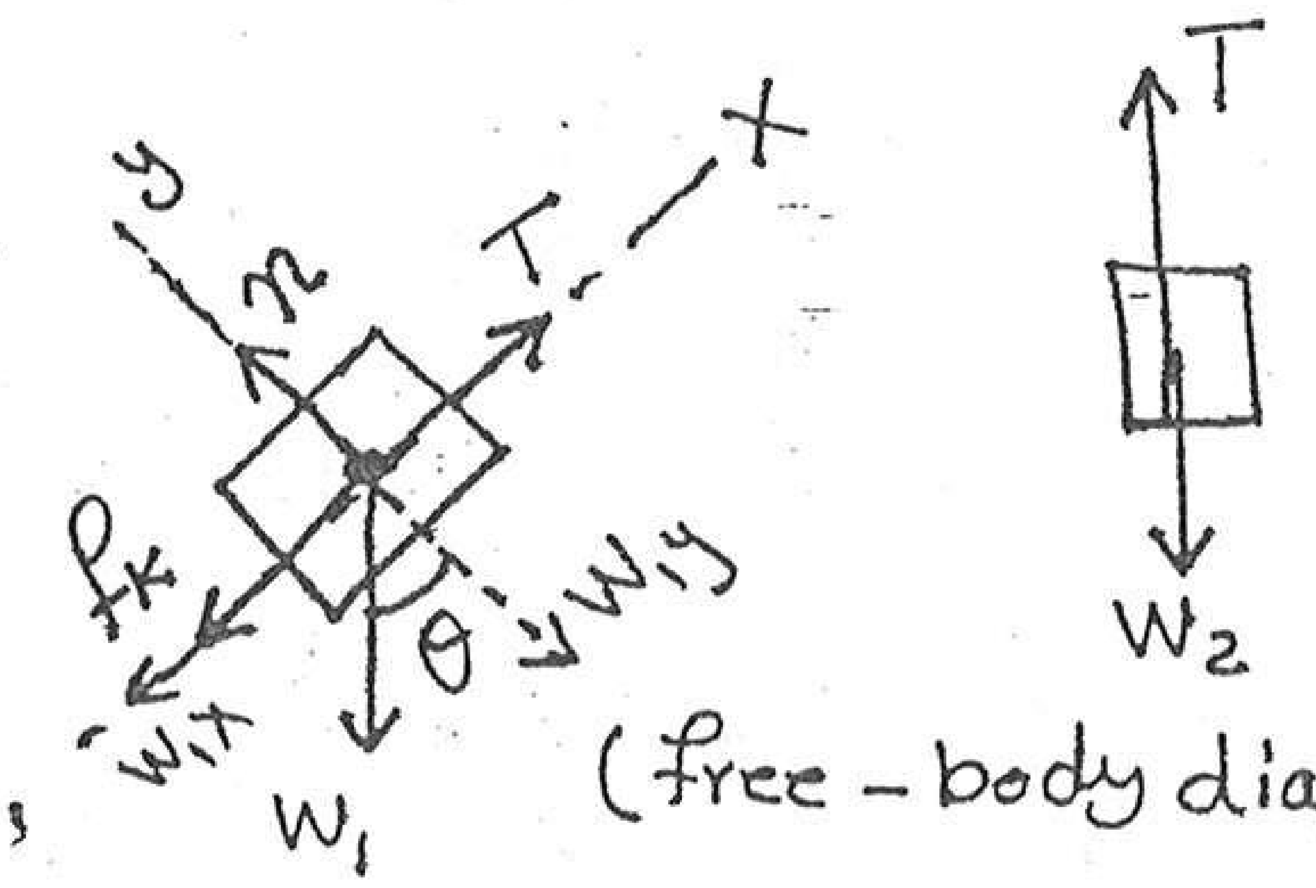
$\therefore$ from (1) and (2) we have: $\mu_k = \dfrac{T}{W_1} = \dfrac{W_2/2}{W_1} = \dfrac{W_2}{2W_1}$

$$\therefore \mu_k = \frac{2}{2(5)} = 0.2$$

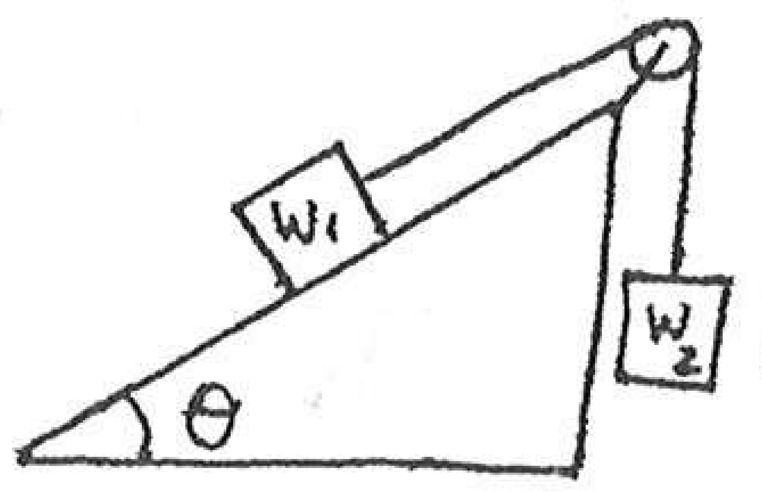
---

ex10: Find the weight W_2 necessary to keep W_1 moving up the plane at constant velocity.

Sol.



(free-body diagram)



Sol. The equilibrium condition for W_1 is:

$$\sum F_x = T - f_k - W_1 \sin\theta = T - \mu_k n - W_1 \sin\theta = 0 \quad \text{—— (1)}$$

$$\sum F_y = n - W_1 \cos\theta = 0 \quad , \text{ Note: } n \neq W_1 \Rightarrow n = W_1 \cos\theta \quad \text{—— (2)}$$

and for W_2:

$$\sum F_x = 0 \; ; \; \sum F_y = T - W_2 = 0 \Rightarrow T = W_2 \quad \text{—— (3)}$$

$\therefore$ equation (1) is: $W_2 - \mu_k W_1 \cos\theta - W_1 \sin\theta = 0$

$$\Rightarrow \boxed{W_2 = \mu_k W_1 \cos\theta + W_1 \sin\theta}$$

H.W: find W_2 if $\theta = 60^\circ$ and $W_1 = 50N$, $\mu_k = 0.3$

* EQUILIBRIUM MOMENT OF A FORCE:

- first condition of equilibrium : $\sum F = 0$
- second condition " " : $\sum \tau = 0$ (τ is moment or torque)

$\tau \curvearrowright - \tau' \curvearrowleft = 0$

* Center of Parallel forces is: (مركز القوى المتوازية)

ويمكن اعتبار ان محصلة القوى تؤثر في نقطة تسمى مركز القوى المتوازية تحدد بمتجه الموضع

$$\vec{r}_c = \frac{\sum_i \vec{r}_i F_i}{\sum_i F_i} = \frac{\vec{r}_1 F_1 + \vec{r}_2 F_2 + \dots}{F_1 + F_2 + \dots} \quad (\vec{r} \text{ Position vector})$$

$$x_c = \frac{\sum_i x_i F_i}{\sum_i F_i}, \quad y_c = \frac{\sum_i y_i F_i}{\sum_i F_i}, \quad z_c = \frac{\sum_i z_i F_i}{\sum_i F_i}$$

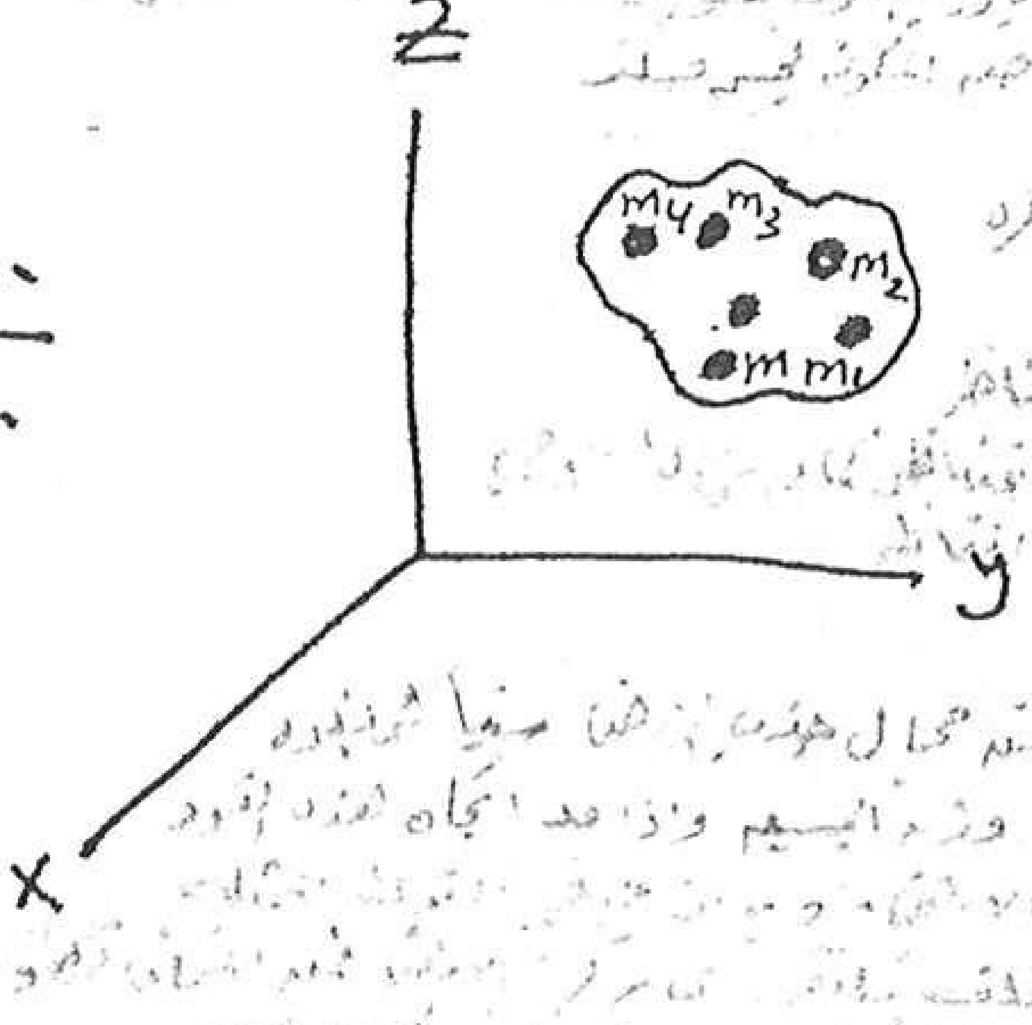
يمكن فصل المعادلة الاتجاهية المتجهة vector الى ثلاث معادلات حيث z_c ، y_c ، x_c هي احداثيات النقطة التي حددت بالمتجه r_c

* Center of mass: the weight of a particle of mass m is: P97

$$F = W = mg$$

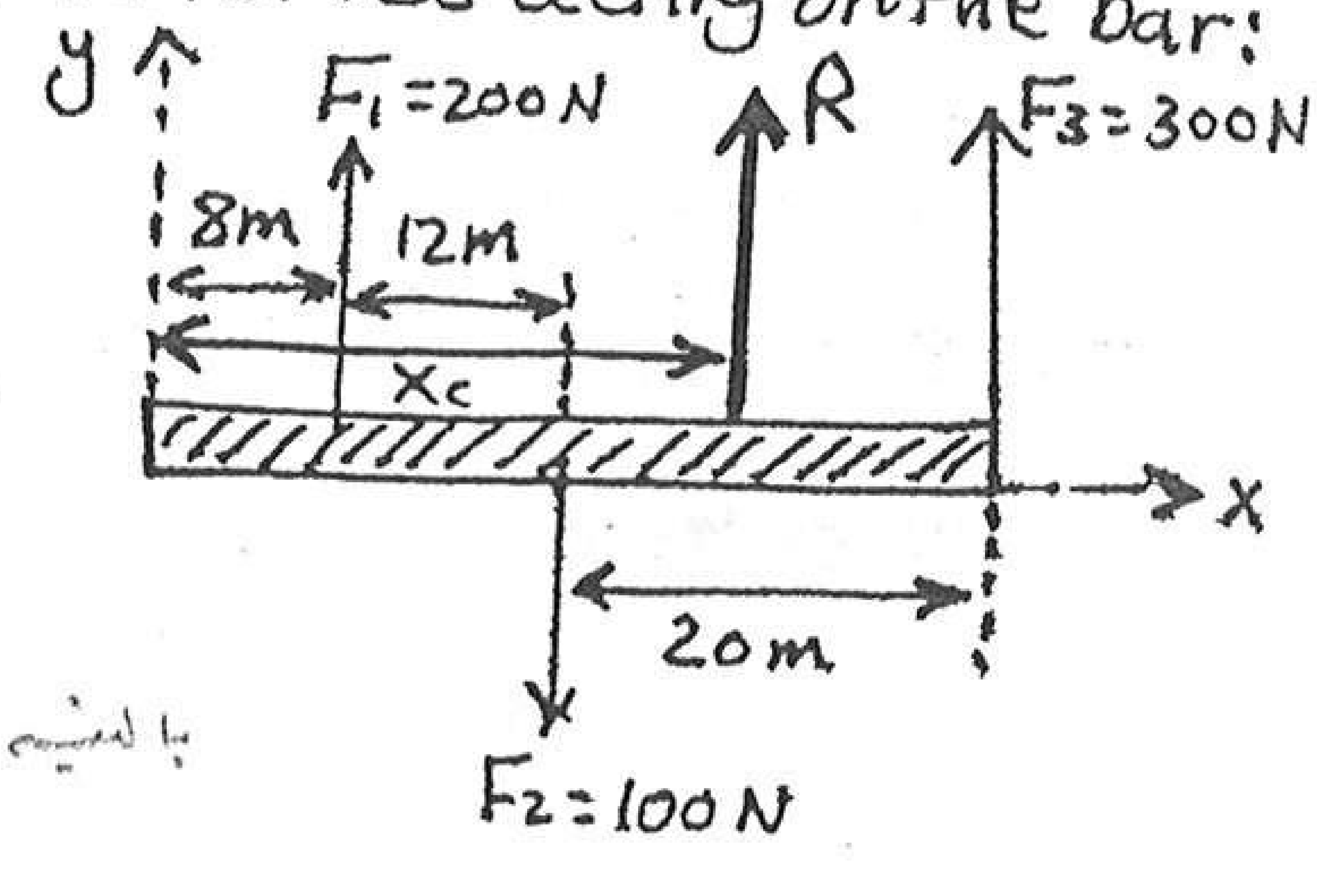
$$\vec{r}_c = \frac{\sum_i \vec{r}_i m_i g}{\sum_i m_i g} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$x_c = \frac{\sum_i m_i x_i}{\sum_i m_i}, \quad y_c = \frac{\sum_i m_i y_i}{\sum_i m_i}, \quad z_c = \frac{\sum_i m_i z_i}{\sum_i m_i}$$



ex1: Find the position for the resultant of the forces acting on the bar:

sol. $R = \sum_i F_i = F_1 - F_2 + F_3 = 200 - 100 + 300$
$= 400\,N$



$$x_c = \frac{\sum_i F_i x_i}{\sum_i F_i} = \frac{(200)(8) + (-100)(20) + (300)(40)}{400}$$

$x_c = 29\,m$

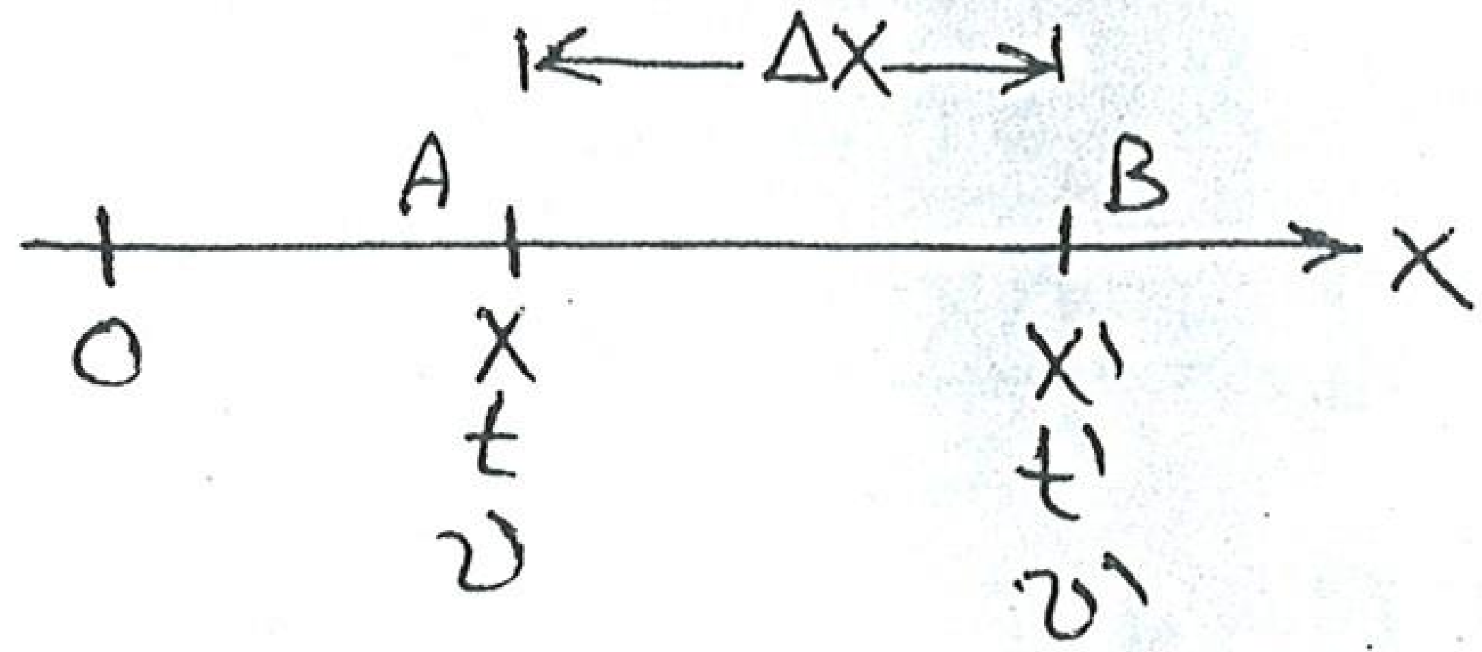
بالنسبة لـ x_i يؤخذ بعد كل قوة عن محور y

Chapter 5 : "Kinematics" علم الحركة المجردة

الحركة في خط مستقيم

* Rectilinear motion:

السرعة Velocity : Average velocity v_{av}:



$$v_{av} = \frac{x' - x}{t' - t} = \frac{\Delta x}{\Delta t}$$

* Instantaneous velocity $v = \lim\limits_{\Delta t \to 0} v_{av} = \lim\limits_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

or $v = \frac{dx}{dt}$ (velocity varies with time $v = f(t)$)

and $dx = v\,dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} v\,dt$

$\Rightarrow x - x_0 = \int_{t_0}^{t} v\,dt \Rightarrow$ Position at any time $\boxed{x = x_0 + \int_{t_0}^{t} v\,dt}$ — (1)

* speed $= |v|$, Unit of velocity and speed: $\frac{m}{sec}$ or $\frac{km}{hr}$

EX1: A particle moves along the x-axis in such away that its
موضع Position at any instant لحظة is given by $x = 5t^2 + 1$, where x
is in meters and t is in seconds. Compute its average velocity متوسط السرعة
in the time interval between (a) 2 sec and 3 sec (b) 2 sec and
2.1 sec, (c) 2 sec and 2.001 sec (d) 2 sec and 2.0001 sec (e) Also
Compute the instantaneous velocity at 2 sec.

Sol. at $t_0 = 2\,sec \Rightarrow x = 5t^2 + 1 \Rightarrow x_0 = 5(2)^2 + 1 = 21\,m$

$\Delta x = X - X_o = X - 21$, $\Delta t = t - t_o = t - 2$

ⓐ for $t = 3\,sec \Rightarrow \Delta t = 3 - 2 = 1\,sec$

and $X = 5t^2 + 1 = 5(3)^2 + 1 = 46\,m \Rightarrow \Delta X = 46 - 21 = 25\,m$

$\therefore v_{av} = \frac{\Delta X}{\Delta t} = \frac{25}{1} = 25\ m/sec$

ⓑ for $t = 2.1\ sec \Rightarrow \Delta t = 2.1 - 2 = 0.1\ sec$

$X = 5(2.1)^2 + 1 = 23.05\,m \Rightarrow \Delta X = 23.05 - 21 = 2.05\,m$

$\therefore v_{av} = \frac{\Delta X}{\Delta t} = \frac{2.05}{0.1} = 20.5\ m/sec$

In the same method:

ⓒ at $t = 2.001\,sec \Rightarrow v_{av} = 20.005\ m/sec$

and ⓓ at $t = 2.0001\,se \Rightarrow v_{av} = 20.00005\,m/sec$

ⓔ The instantaneous velocity at $t = 2\,sec$:

$v = \frac{dX}{dt} = \frac{d}{dt}(5t^2 + 1) = 10t$

$v\Big|_{t=2sec} = 10(2) = 20\,m/sec$

<u>Note</u>: The instantaneous velocity equal average velocity when Δt approaches zero.

*Acceleration:

average acceleration : $a_{av} = \dfrac{v' - v}{t' - t} = \dfrac{\Delta v}{\Delta t} \left(\dfrac{m}{sec^2}\right)$

instantanous acceleration: $a = \lim\limits_{\Delta t \to 0} a_{av} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$

and $a = \dfrac{dv}{dt}$

the velocity at any time t: $\int_{v_0}^{v} dv = \int_{t_0}^{t} a\, dt$

$$\Rightarrow v - v_0 = \int_{t_0}^{t} a\, dt \Rightarrow \boxed{v = v_0 + \int_{t_0}^{t} a\, dt} \quad — (2)$$

ex2: A body moves along the X-axis according to the law: (حسب العلاقة)
$X = 2t^3 + 5t^2 + 5$, where x in meters and t in seconds,
find (a) The velocity and the acceleration at any time (b) the position, velocity and acceleration at $t = 2sec$, and (c) the average velocity and acceleration between $t = 2sec$ and $t = 3sec$.

Sol. (a) $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(2t^3 + 5t^2 + 5) = (6t^2 + 10t)$

$a = \dfrac{dv}{dt} = \dfrac{d}{dt}(6t^2 + 10t) = (12t + 10)$

(b) At $t = 2sec \Rightarrow X = 2(2)^3 + 5(2)^2 + 5 = 41\ m$

$v = 6(2)^2 + 10(2) = 44\ m/sec$

$a = 12(2) + 10 = 34\ m/sec^2$

$t=3sec$ $v = \dfrac{dx}{dt} = 6t^2 + 10$

$t=3sec$ $v = 6(3)^2 + 10(3) = 84$

$v_0 = 6(2)^2 + 10(2) = 44$

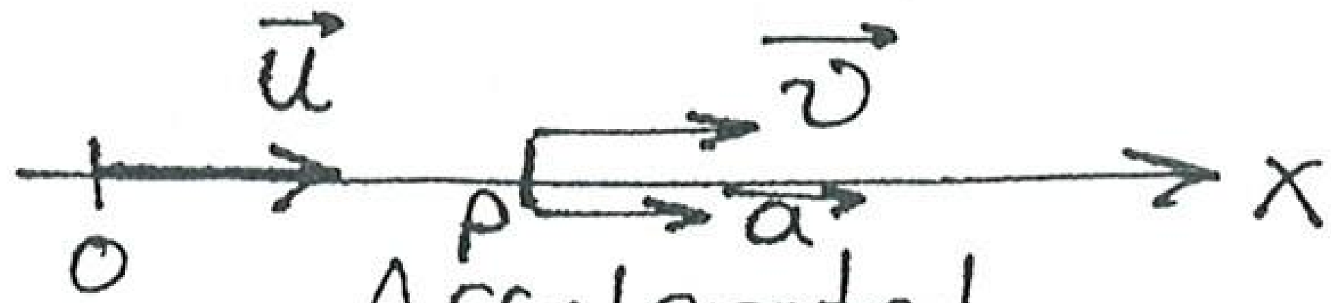
$\Delta v = 84 - 44 = 40$

(c) $\Delta t = 3 - 2 = 1\ sec$, $\Delta X = X - X_0$, $\Delta v = v - v_0$

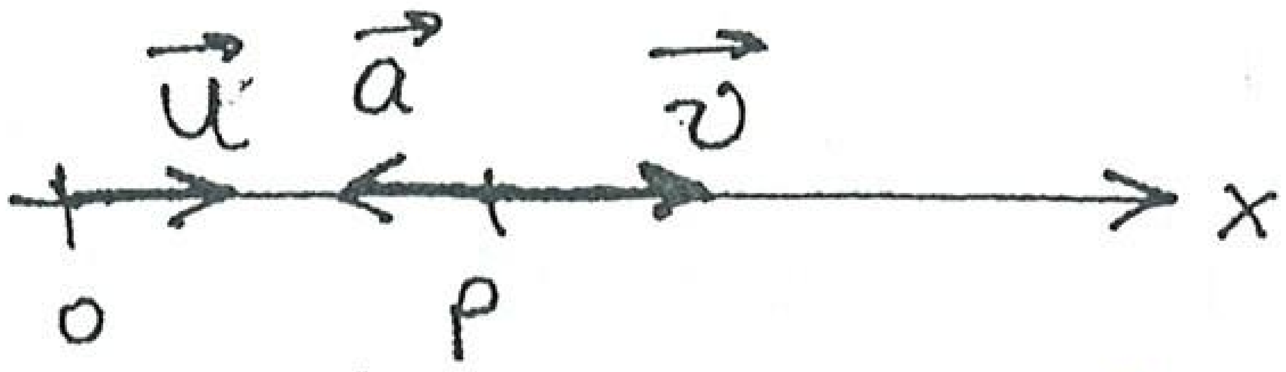
$X|_{t=2sec} = 2(2)^3 + 5(2)^2 + 5 = 41 m$, $X|_{t=3sec} = 2(3)^3 + 5(3)^2 + 5 = 104 m$

$\Delta X = 104 - 41 = 63m \Rightarrow v_{av} = \dfrac{\Delta X}{\Delta t} = \dfrac{63}{1} = 63\ m/sec$, $a_{av} = \dfrac{\overset{v_2 - v_1}{\Delta v}}{\Delta t} = \dfrac{40}{1 \ldots}$

Note: The vectors of velocity and acceleration in Rectilinear motion have the same direction if the Body Accelerated and opposite direction if the body Retarded.



Accelerated



Retarded تعاكس

*** Some special motions:**

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = v\int_{t_0}^{t} dt$$
$$x - x_0 = v(t - t_0)$$
$$x = x_0 + v(t - t_0)$$

الحركة المنتظمة في خط مستقيم

Ⓐ Uniform rectilinear motion:

v: constant

$$a = \frac{dv}{dt} = 0 \quad \Rightarrow x = x_0 + \int_{t_0}^{t} v\,dt = x_0 + v\int_{t_0}^{t} dt$$

or $$\boxed{x = x_0 + v(t - t_0)} \quad \text{— (3)}$$

الحركة ذات التعجيل المنتظم في خط مستقيم

ⓑ Uniformly accelerated rectilinear motion:

a: Constant

$$v = v_0 + \int_{t_0}^{t} a\,dt = v_0 + a\int_{t_0}^{t} dt \quad \text{or} \quad \boxed{v = v_0 + a(t - t_0)} \text{— (4)}$$

from equation (1) $$\Rightarrow x = x_0 + \int_{t_0}^{t} [v_0 + a(t - t_0)]\,dt$$

$$\Rightarrow x = x_0 + v_0\int_{t_0}^{t} dt + a\int_{t_0}^{t} (t - t_0)\,dt \quad \text{or}$$

$$\int_{x_1}^{x_2} x^{n} = \frac{1}{n+1} x^{n+1}\Big|_{x_1}^{x_2}$$

$$\boxed{x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2} \quad \text{— (5)}$$

from (4) we have: $t-t_0 = \dfrac{v-v_0}{a}$ عوض substitution in (5)

$$\Rightarrow X-X_0 = v_0\left(\frac{v-v_0}{a}\right)+\frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2$$

$$a(X-X_0) = v_0 v - v_0^2 + \frac{1}{2}\left(v^2 - 2vv_0 + v_0^2\right)$$

$$a(X-X_0) = \cancel{v_0 v} - v_0^2 + \frac{1}{2}v^2 - \cancel{vv_0} + \frac{1}{2}v_0^2$$

$$-v_0^2+\frac{1}{2}v_0^2 = -\frac{1}{2}v_0^2$$

$$a(X-X_0) = -\frac{1}{2}v_0^2 + \frac{1}{2}v^2 \Rightarrow \boxed{v^2 = v_0^2 + 2a(X-X_0)} \quad (6)$$

If $X_0=0$, $t_0=0$:

$X = v\cdot t$ — (3)' ; $v = v_0 + at$ — (4)'

$X = v_0 t + \frac{1}{2}at^2$ — (5)' ; $v^2 = v_0^2 + 2aX$ — (6)'

الحركة العمودية الطليقة تحت تأثير الجاذبية

(c) Free Vertical motion under the action of gravity:

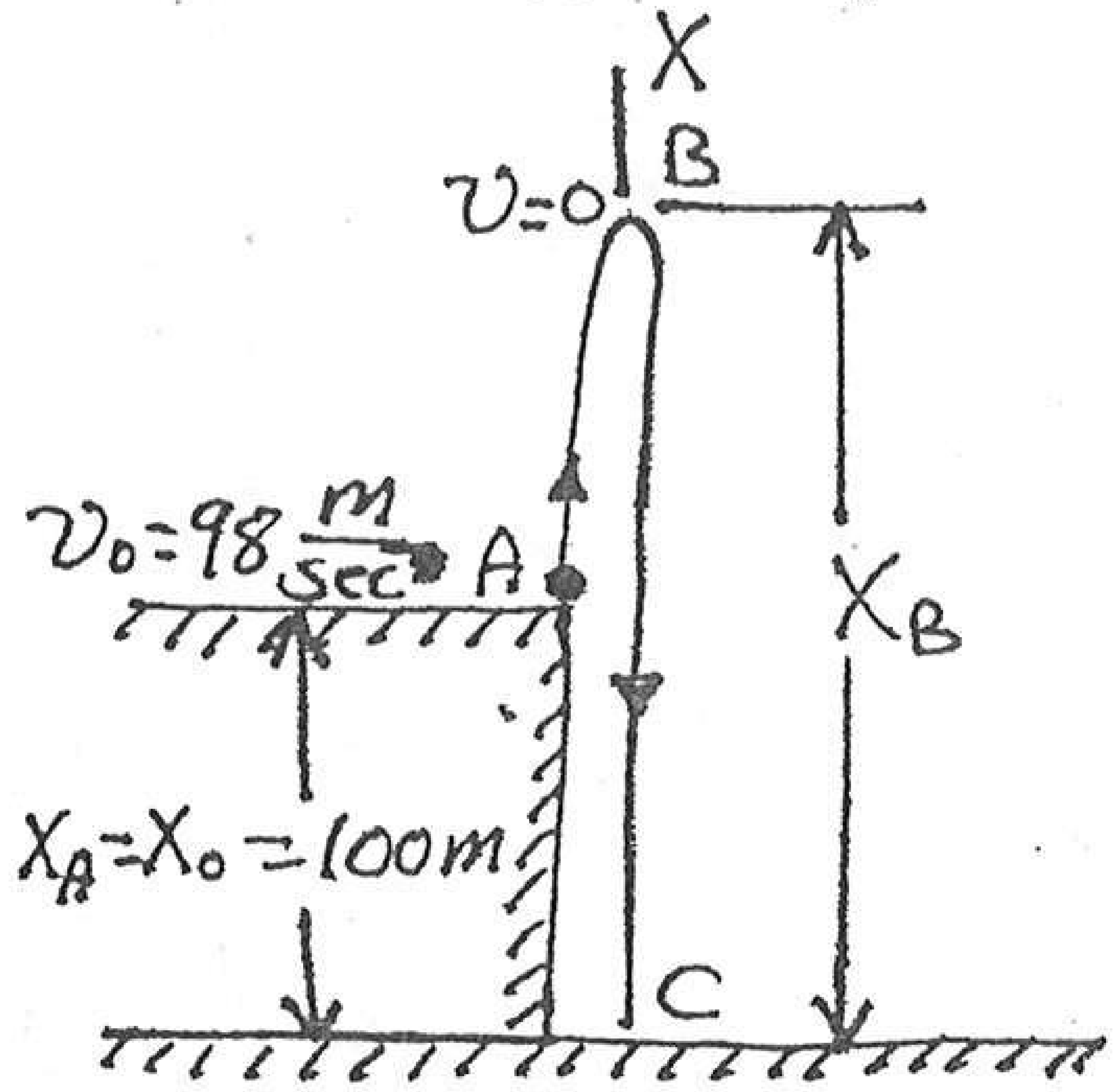
This case of uniformly accelerated motion.

$a = -g$ (gravitational acceleration)

- Summary of Relations for Rectilinear motion:

		at $X_0=0$, $t_0=0$
Uniform rectilinear motion	$a=0$ $v=$ constant $X = X_0 + v(t-t_0)$	$X = vt$
Uniformly accelerated motion	$a=$ constant $v = v_0 + a(t-t_0)$ $X = X_0 + v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2$	$v = v_0 + at$ $X = v_0 t + \frac{1}{2}at^2$
الحركة العمودية الطليقة Free vertical motion	$a = -g = -9.8\ m/sec^2$ $v = v_0 - g(t-t_0)$ $y = y_0 + v_0(t-t_0) - \frac{1}{2}g(t-t_0)^2$	$v = v_0 - gt$ $y = v_0 t - \frac{1}{2}gt^2$

ex3 : A bullet (رصاص) is fired straight upward with a velocity of 98 m/sec from the top of a building 100 m high. Find ⓐ its maximum height above the ground, ⓑ the time required to reach it ⓒ the velocity it has when it reaches the ground, and ⓓ the total time which elapses (تنقضي) before the bullet reaches the ground.



Sol: $t_0=0,\ v_0=98\ \frac{m}{sec^2}$

$X_0=X_A=100\ m\ ,\ a=-9.8\ \frac{m}{sec^2}$

$v=v_0-gt$

$v=98-9.8t$ ---- (1)

and $X=X_0+v_0t-\frac{1}{2}g(t-t_0)^2$

$X=100+98t-\frac{9.8}{2}t^2 \Rightarrow X=100+98t-4.9t^2$ ---- (2)

ⓐ at maximum height $v=0$; عند اعلى ارتفاع $v=0$

$0=98-9.8t \Rightarrow t=10\,sec$ ← بتعويض $v=0$ في (1) نحصل على قيمة t اللازمة لبلوغ اعلى ارتفاع

ⓑ $X_B=100+98(10)-4.9(10)^2=590\,m$ بتعويض قيمة t في (2) نحصل على اعلى ارتفاع X_B نقطة X فوق

ⓒ The time required to reach the ground ; $X_C=0$ ← يجاد الزمن اللازم للوصول للارض عند C بتعويض عن $x=0$ في (2) حيث من نقطة بداية الاحداثيات عندها $x=0$

$0=100+98t-4.9t^2 \Rightarrow \underbrace{t=-0.96\,sec}_{\text{negligeable}}\ ,\ t=20.96\,sec$

$v_C=98-9.8(20.96)=-107.41\ m/sec$

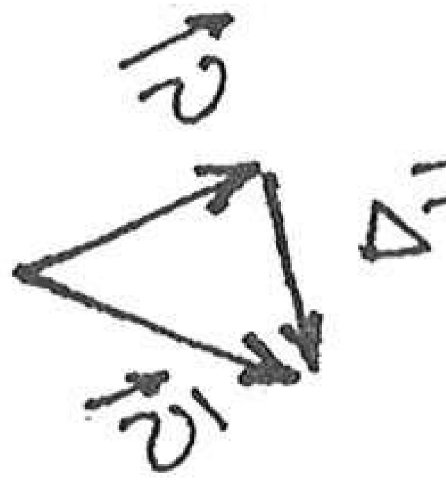
or By: $v^2=$

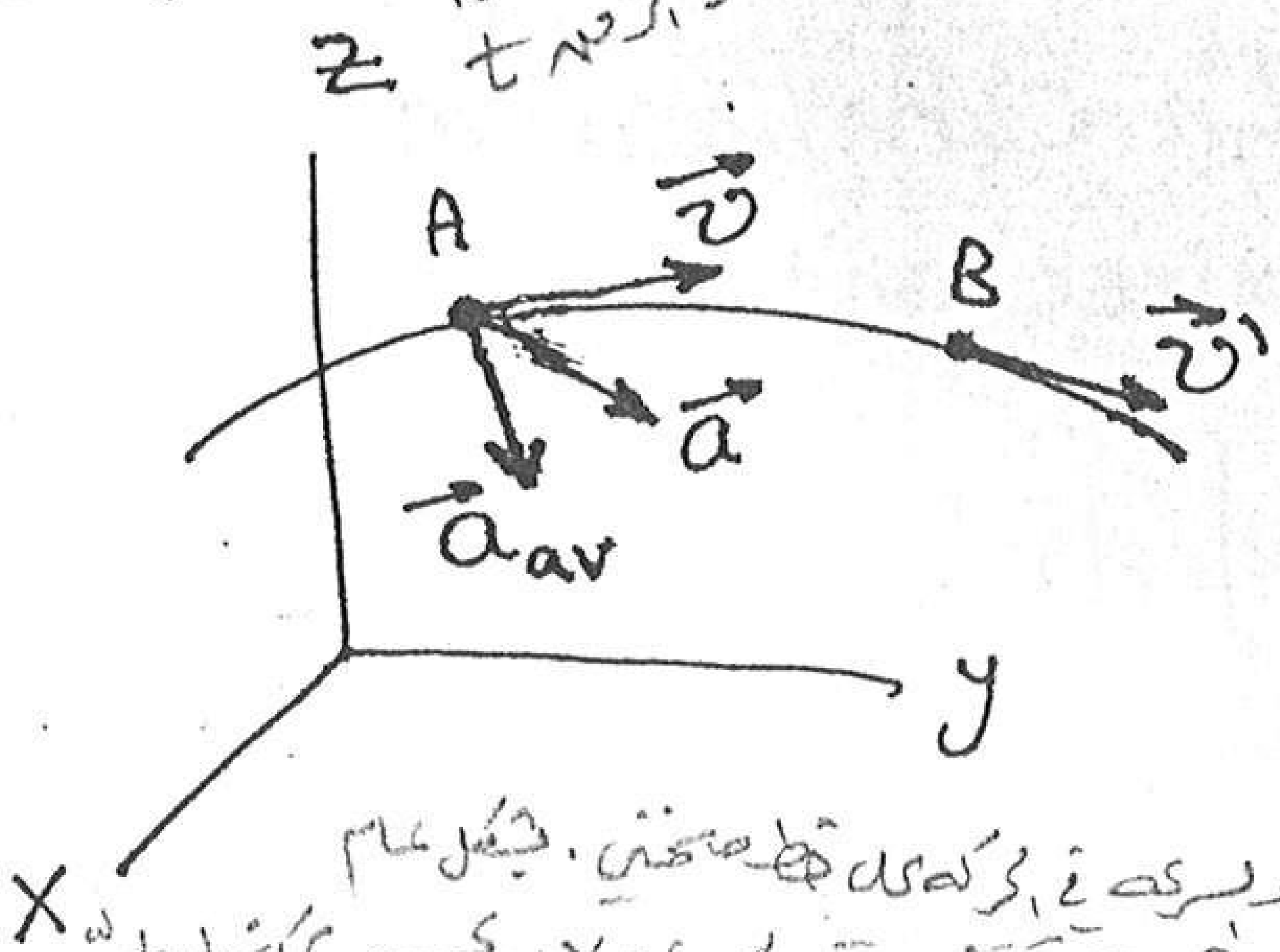
الحركة على خط منحني

* Curvilinear motion : acceleration:

الجسم يتواجد عند النقطة A في الزمن t — وتتحول للنقطة B في الزمن t'

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t}$$





$\vec{a}_{av}$ Parallel to $\Delta\vec{v}$

- instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{ave} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

تتغير السرعة في الحركة على خط منحني بشكل عام قيمة واتجاها تتغير قيمة السرعة لان الجسم يمكن ان يسرع او يخفض سرعته والمسار يتغير باستمرار والسرعة هي مماس للمسار لذا فاتجاهها يتغير

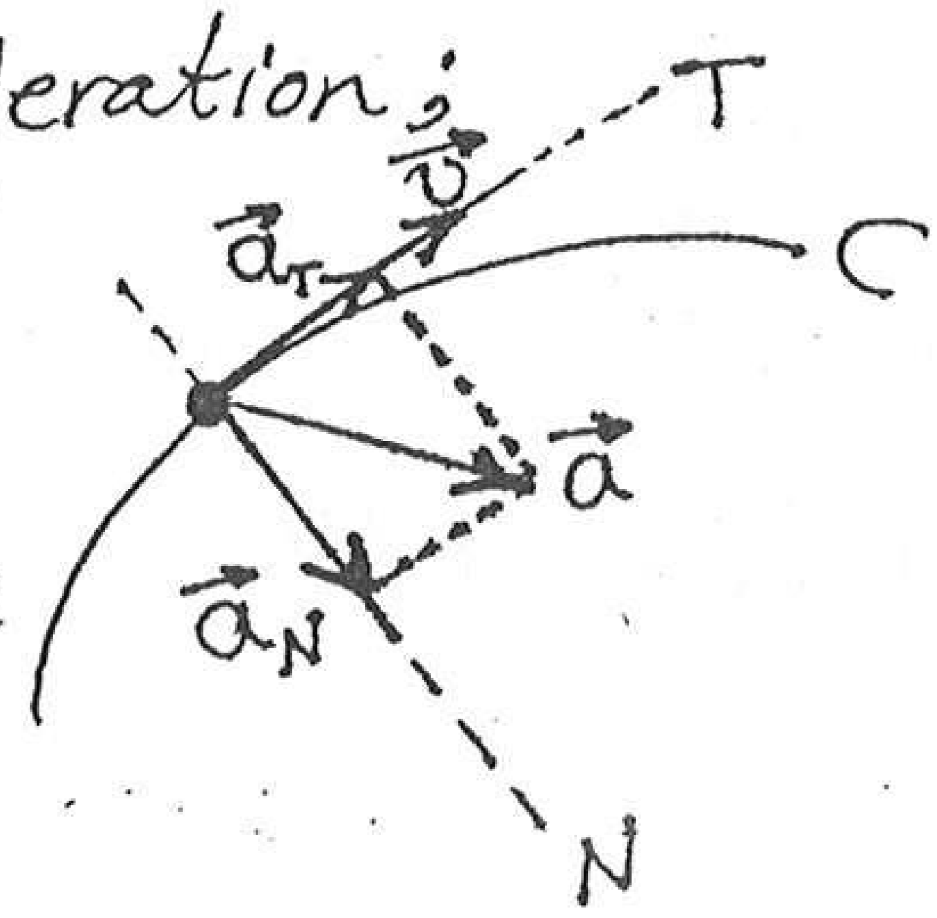
or $\vec{a} = \frac{d\vec{v}}{dt}$; $\vec{v} = v_x i + v_y j + v_z K$

and $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$

* We may decompose (تحليل) the acceleration into a tangential component $\vec{a}_T$, Parallel to the tangent (موازية لمماس) Called tangential acceleration (التعجيل المماسي) and a normal component $\vec{a}_N$ Parallel to the normal (موازية للعمود) Called normal acceleration or centripetal acceleration:

التعجيل المركزي او العمودي

$$\boxed{\vec{a} = \vec{a}_T + \vec{a}_N}$$



The magnitude of the tangential acceleration is: $\boxed{a_T = \frac{dv}{dt}}$,

اذا كانت الحركة مستقيمة (قيمة السرعة ثابتة) فان $a_T = 0$ لذلك ليس هناك تعجيل مماسي

and the magnitude of the normal or centripetal acceleration:

$$\boxed{a_N = \frac{v^2}{R}} \quad ; \; R: \text{radius of curvature of the Path.}$$

نصف قطر تكور المسار

واذا كانت الحركة على خط مستقيم (اي ان اتجاه الحركة لا يتغير) فان نصف قطر التكور $R = \infty$ لذلك $a_N = 0$ اي ليس هناك له تعجيل مركزي

التغير في قيمة السرعة : التعجيل المماسي
التغير في اتجاه السرعة : التعجيل العمودي

الحركة بتعجيل ثابت　5-8

* Motion with constant acceleration:

In curvilinear motion, a: constant in magnitude and direction:　$a=\frac{dv}{dt}$

$$\int_{v_0}^{v} d\vec{v} = \int_{t_0}^{t} \vec{a}\,dt = \vec{a}\int_{t_0}^{t} dt \;, \Rightarrow \vec{v}-\vec{v}_0 = \vec{a}(t-t_0)$$

and $\boxed{\vec{v} = \vec{v}_0 + \vec{a}(t-t_0)}$

(Velocity at any time)

and $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int_{r_0}^{r} d\vec{r} = \int_{t_0}^{t} [\vec{v}_0 + \vec{a}(t-t_0)]\,dt$,　$\int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} a(t-t_0)\,dt$

$v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2$

$$\Rightarrow \vec{r}-\vec{r}_0 = \vec{v}_0(t-t_0) + \frac{1}{2}\vec{a}(t-t_0)^2$$

$$\therefore \boxed{\vec{r} = \vec{r}_0 + \vec{v}_0(t-t_0) + \frac{1}{2}\vec{a}(t-t_0)^2}$$

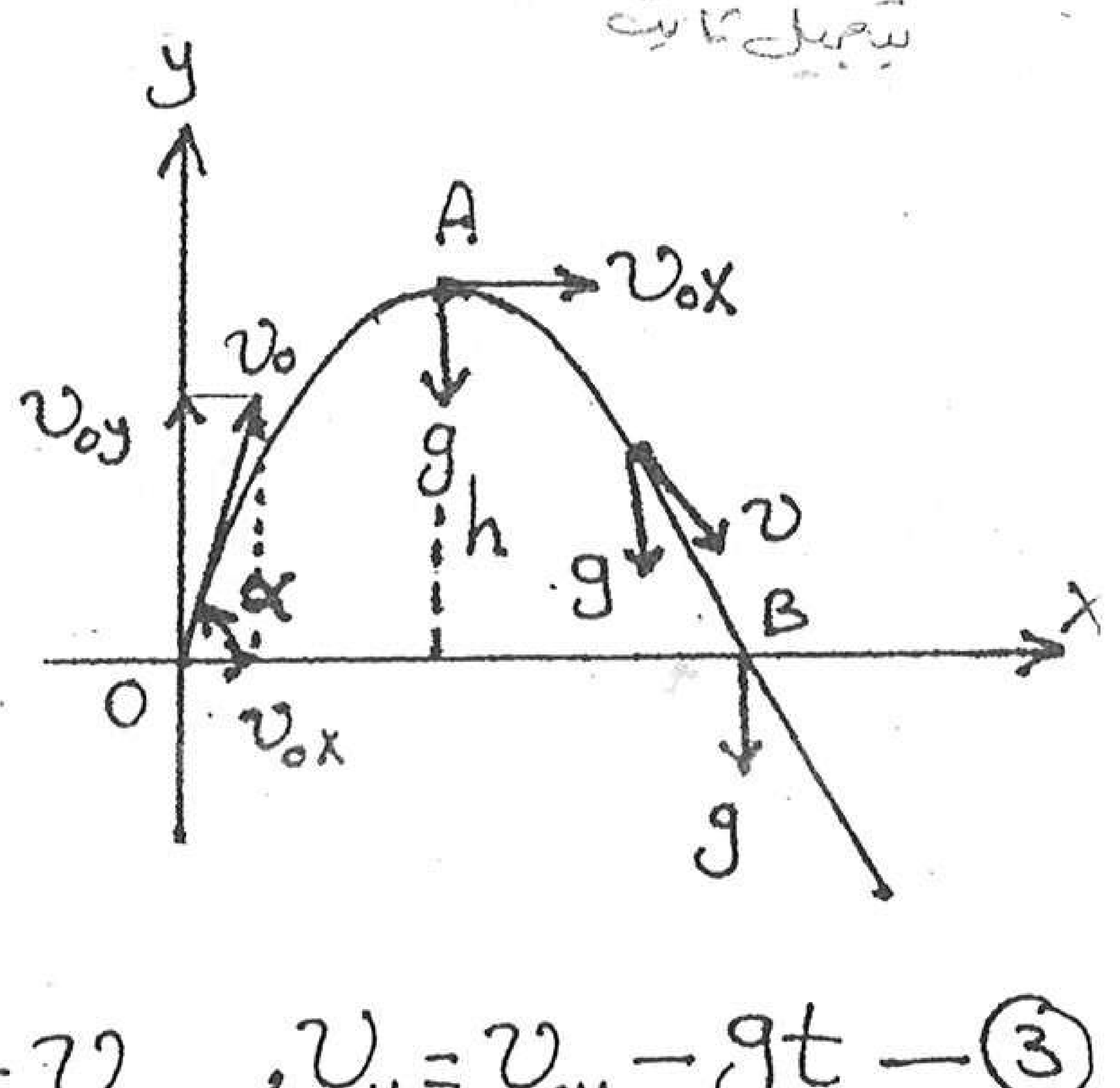
هذه المعادلة تمثل الموضع لأي لحظة

(Position at any time)

حركة المقذوف واحدة من أهم الأمثلة لحركة بتعجيل ثابت

حركة المقذوف

Projectile Motion: In x-y-plane



$a=-g$, α: is the angle of $\vec{v}_0$ with x-axis

$v_{0x} = v_0 \cos\alpha$ — (1)

$v_{0y} = v_0 \sin\alpha$ — (2)

and the velocity at any time: $v_x = v_{0x}$, $v_y = v_{0y} - gt$ — (3)

The magnitude of the velocity at any time is:

السرعة x تبقى ثابتة وهو ما يعني انه لا يوجد تعجيل بهذا الاتجاه

$$v = \sqrt{v_x^2 + v_y^2}$$

The coordinates of the Particles $x = v_{0x} \cdot t$, $y = v_{0y}t - \frac{1}{2}gt^2$ — (4)

مشتقة من معادلة الموضع ٢ بعد وضع $r_0=0$ و $t_0=0$

5-9

الزمن اللازم لبلوغ القذيفة أعلى نقطة

The time required for the Projectile to reach the highest Point, at A ; $v_y = 0$; from (3) we have:

حيث ان سرعة القذيفة في تلك النقطة تكون افقية

$$t = \frac{v_{oy}}{g} \quad \text{or} \quad t = \frac{v_o \sin\alpha}{g}, \quad \text{substituting in eq. (4)}$$

$$\text{at } A ; y = h \Rightarrow h = v_o \sin\alpha \left(\frac{v_o \sin\alpha}{g}\right) - \frac{1}{2} g \left(\frac{v_o \sin\alpha}{g}\right)^2$$

$$h = \frac{v_o^2 \sin^2\alpha}{g} - \frac{1}{2}\frac{v_o^2 \sin^2\alpha}{g} \Rightarrow \boxed{h = \frac{v_o^2 \sin^2\alpha}{2g}}$$

حفظ

The time required for the Projectile to return to ground level at B called the <u>time of flight</u> : $y = 0$

The equation (4) become: $0 = v_o \sin\alpha \, t - \frac{1}{2} g t^2$

$$\text{either } t = 0 \quad \text{or} \quad \boxed{t_f = \frac{2 v_o \sin\alpha}{g}}$$

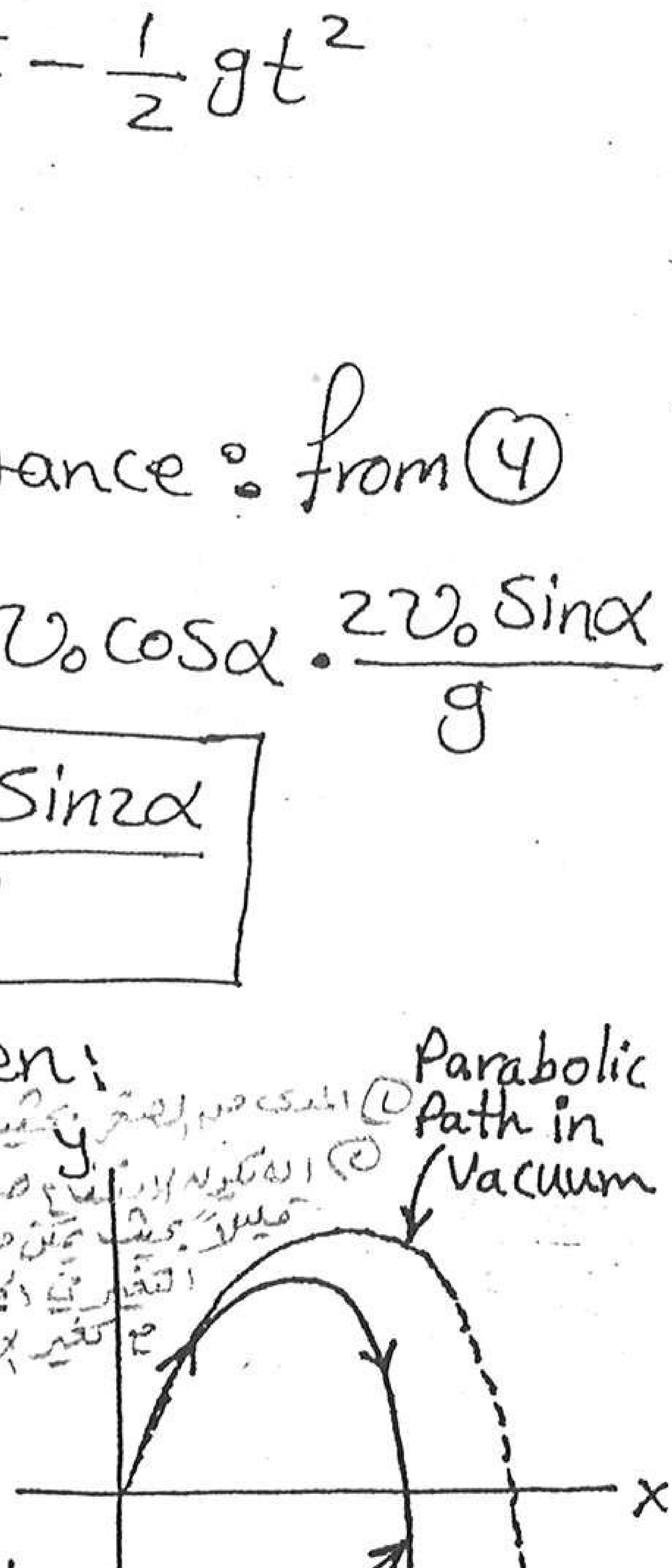
The Range is the total horizontal distance: from (4)

$$X = v_{ox} t \Rightarrow R = v_o \cos\alpha \, t_f \Rightarrow R = v_o \cos\alpha \cdot \frac{2 v_o \sin\alpha}{g}$$

$$\Rightarrow R = v_o^2 \frac{2 \sin\alpha \cos\alpha}{g} \Rightarrow \boxed{R = \frac{v_o^2 \sin 2\alpha}{g}}$$

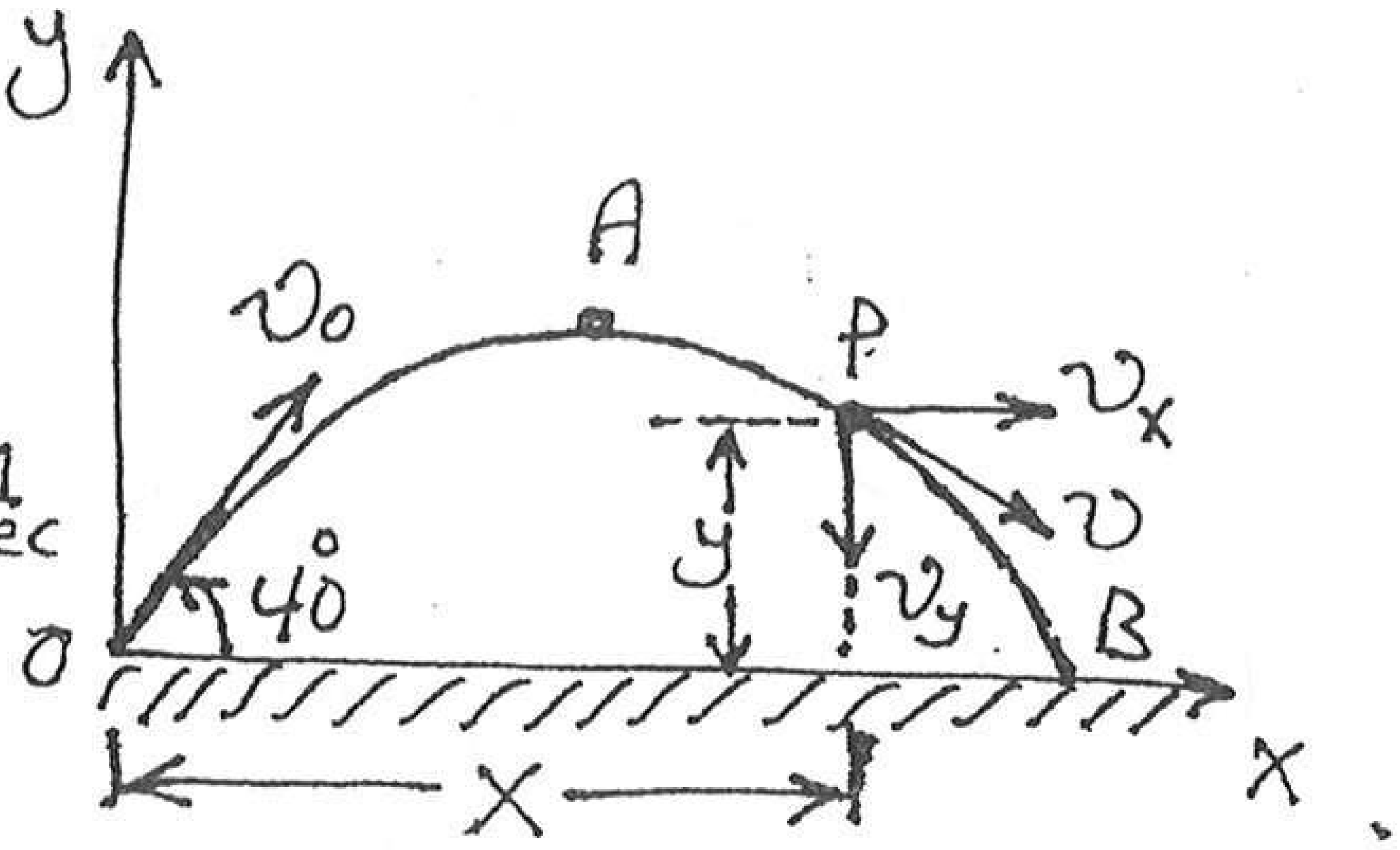
The results we have obtained are valid when:

1. The range is small enough so that the curvature of the earth may be neglected.
2. The altitude is small enough so that the variation of gravity with height may be neglected.
3. The initial velocity is small enough so that air resistance may be neglected.



تطلق بندقية رصاصة

ex: A gun fires a bullet with a velocity of 200 m/sec at an angle of 40° with the ground (الأفق). Find the Velocity and Position of the bullet after 20 sec. Also find the range and the time required for the bullet to return to ground.



Sol: $v_o = 200\ m/sec$

$\alpha = 40°$

$\Rightarrow v_{ox} = v_o \cos\alpha = 200 \cos 40 = 153.2\ \frac{m}{sec}$

$v_{oy} = v_o \sin\alpha = 128.6\ m/sec$

The Velocity at any time:

$v_x = v_{ox} = 153.2\ m/sec$; $v_y = v_{oy} - gt = 128.6 - 9.8t$

and the Coordinates of the bullet at any time: واحداثيات الرصاصة في أي لحظة

$X = v_{ox} \cdot t = 153.2t$; $y = v_{oy}t - \frac{1}{2}gt^2 = 128.6t - \frac{1}{2}(9.8)t^2$

and for $t = 20$ sec $v_y = 128.6 - 9.8(20) = 128 - 196 = -67.4$

$\Rightarrow v_x = 153.2\ m/sec$, $v_y = -67.4\ \frac{m}{sec}$ (v_y is negative means that the bullet is descending (هبوط).

The velocity $v = \sqrt{v_x^2 + v_y^2} = 167.4\ \frac{m}{sec}$

The Position: $X = 153.2(20) = 3064\ m$; $y = 128.6(20) - \frac{1}{2}(9.8)(20)^2$

$\Rightarrow y = 612\ m$

and the height at A: $h = \frac{v_o^2 \sin^2\alpha}{2g} = \frac{(200)^2(\sin 40)^2}{2(9.8)} = 843.7\ m$

the range: $R = \frac{v_o^2 \sin 2\alpha}{g} = \frac{(200)^2 \sin 80}{9.8} = 4021\ m$

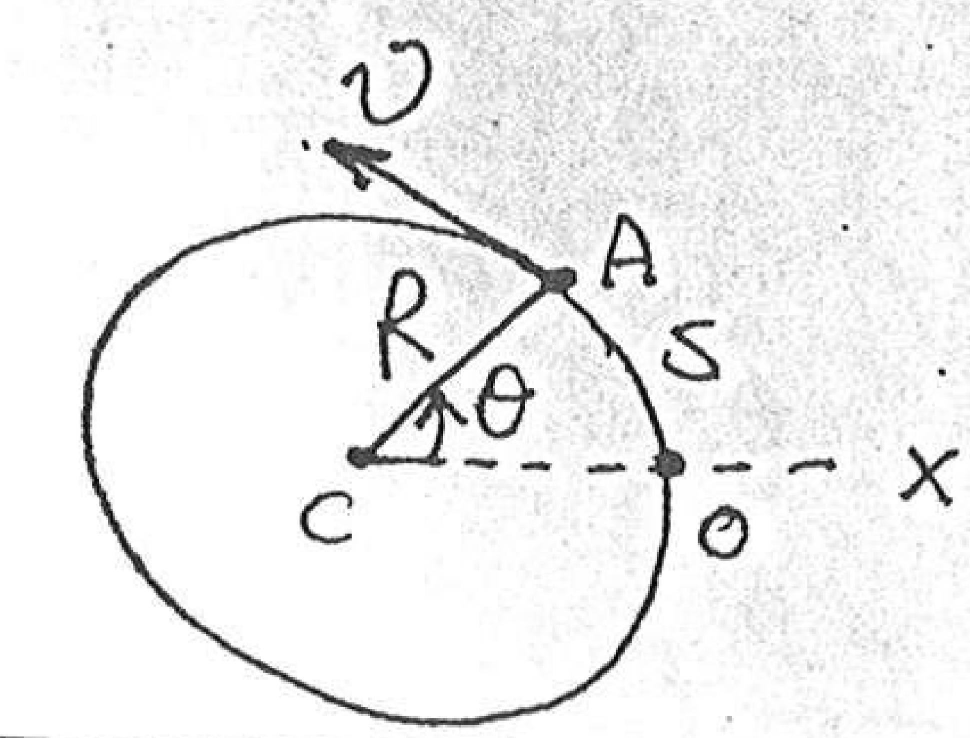
The time required to go from O: $t_f = \frac{2v_o \sin\alpha}{g} = \frac{2(200)\sin 40}{9.8}$

مقدار الزاوية بنصف قطرية = طول القوس / نصف القطر

محيط الدائرة = $2\pi R$

الزاوية الدورية كاملة بنصف قطرية = $\frac{2\pi R}{R} = 2\pi\,rad$

☆ Circular Motion : Angular velocity



The velocity v being tangent to the Circle and Perpendicular to the radius R :

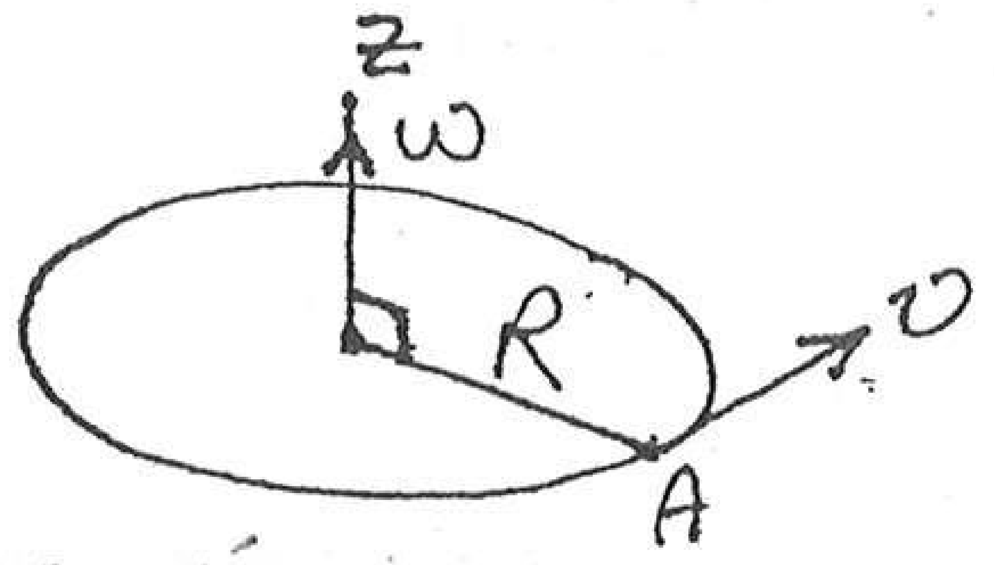
$$\theta_{(rad)} = \frac{S}{R} \Rightarrow S = R\theta$$

$$v = \frac{ds}{dt} = \frac{d}{dt}(R\theta) = \theta\cancel{\frac{dR}{dt}}^{0} + R\frac{d\theta}{dt} \Rightarrow \boxed{v = R\frac{d\theta}{dt}}$$

سرعة زاوية

$$\Rightarrow \text{angular velocity } \omega = \frac{d\theta}{dt}\left(\frac{rad}{sec}\right)$$

$$\Rightarrow \boxed{v = \omega R}$$



اتجاه ω عمودي على مستوى الحركة

direction of ω is Perpendicular to the Plane of motion

Note : ω = constant in uniform circular motion; the motion is Periodic and the Particle passes through each Point of the Circle at regular (منتظم) intervals of time.

مدة الذبذبة

P ; is the Period , the time required for a complete turn or revolution, and the frequency ν is the number of revolution per Unit time.

☆ If the Particle makes n revolution in time t ;

$$\boxed{\text{The Period } P = \frac{t}{n}} \ (sec)$$

$$\boxed{\text{frequency } \nu = \frac{n}{t} = \frac{1}{P}} \ (sec^{-1} = \text{Hertz (Hz)})$$

(rPs) revolution Per second = Hz

(rPm) " " minute = $(minute)^{-1} = \frac{1}{60}$ Hz

- ~~If~~ ω constant:

$$\int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} \omega\, dt = \omega \int_{t_0}^{t} dt \quad \text{or} \quad \boxed{\theta = \theta_0 + \omega(t - t_0)}$$

If $\theta_0 = 0$ and $t_0 = 0 \Rightarrow \theta = \omega t$ or $\omega = \dfrac{\theta}{t}$

and for complete revolution, $t = P$, $\theta = 2\pi$

$$\Rightarrow \boxed{\omega = \frac{2\pi}{P} = 2\pi\nu}$$

* Circular motion: Angular acceleration

angular acceleration: $\boxed{\alpha = \dfrac{d\omega}{dt}} = \dfrac{d}{dt}\left(\dfrac{d\theta}{dt}\right) = \dfrac{d^2\theta}{dt^2} \quad \left(\dfrac{rad}{sec^2}\right)$

α: constant when the circular motion is uniformly accelerated:

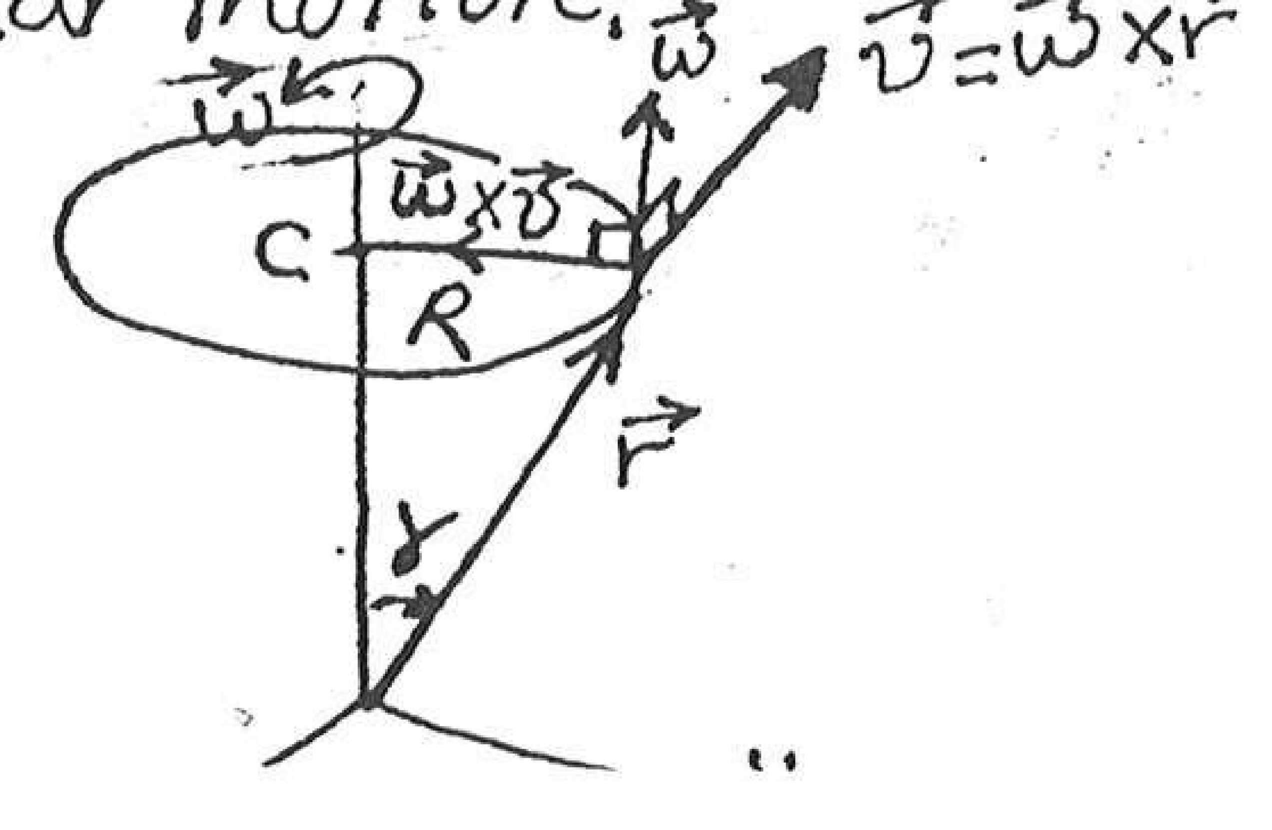
$$\int_{\omega_0}^{\omega} d\omega = \int_{t_0}^{t} \alpha\, dt = \alpha \int_{t_0}^{t} dt \quad \underline{\text{or}} \quad \boxed{\omega = \omega_0 + \alpha(t - t_0)}$$

$$\therefore \omega = \frac{d\theta}{dt} = \omega_0 + \alpha(t - t_0) \Rightarrow \int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} \omega_0\, dt + \alpha \int_{t_0}^{t} (t - t_0)\, dt$$

$$\Rightarrow \boxed{\theta = \theta_0 + \omega_0(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2}$$

when $t_0 = 0$, $\theta_0 = 0 \Rightarrow \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

* Velocity and acceleration in circular motion:



$R = r\sin\gamma$ and $v = \omega R$

$\therefore v = \omega r \sin\gamma$

$$\Rightarrow \boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

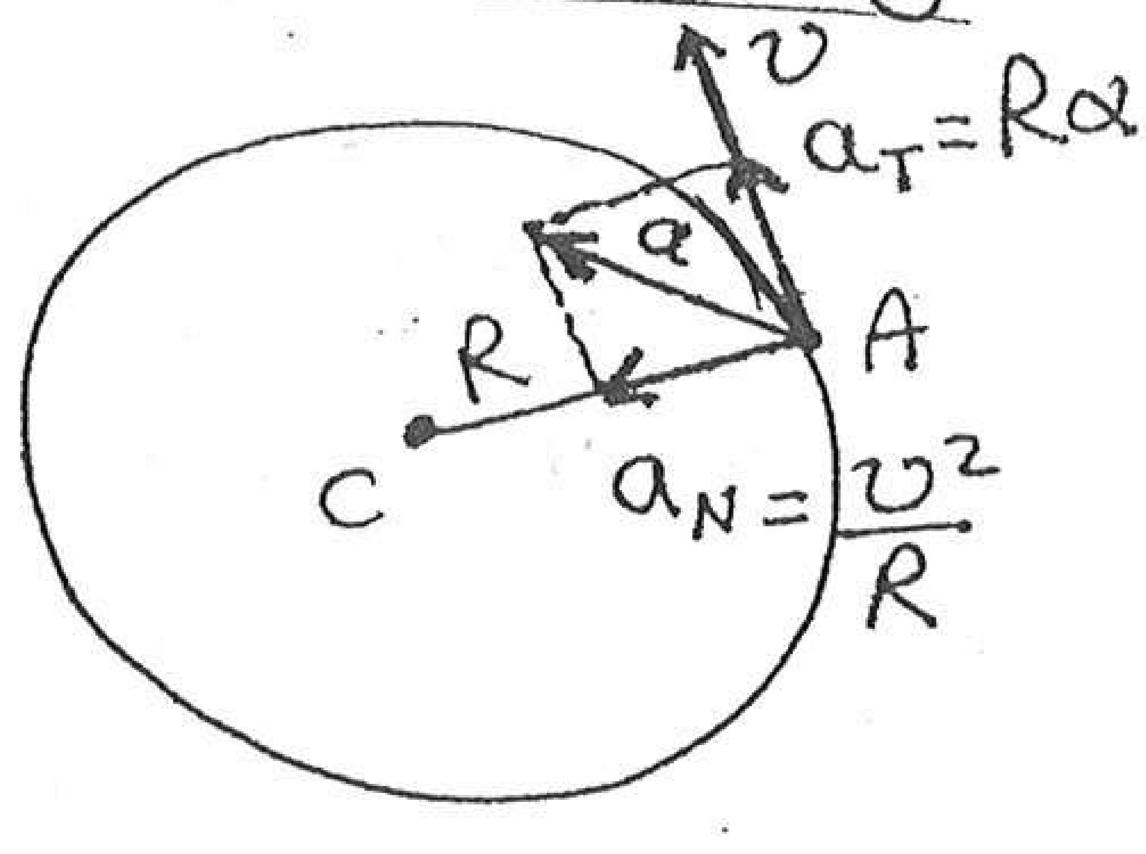
and $\vec{a} = \frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \cancel{\frac{d\vec{\omega}}{dt}} \times \vec{r}$; $\vec{\omega}$: constant

$$\Rightarrow \vec{a} = \vec{\omega} \times \frac{d\vec{r}}{dt} \Rightarrow \boxed{\vec{a} = \vec{\omega} \times \vec{v}}$$

$$\because \vec{v} = \vec{\omega} \times \vec{r}$$

$$\Rightarrow \boxed{\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r})}$$

Note : in Uniform Circular motion the acceleration is باتجاه القطر Perpendicular to the velocity and يشير Points radially الى الداخل inward.



The Centripetal acceleration:

$$a_N = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R$$

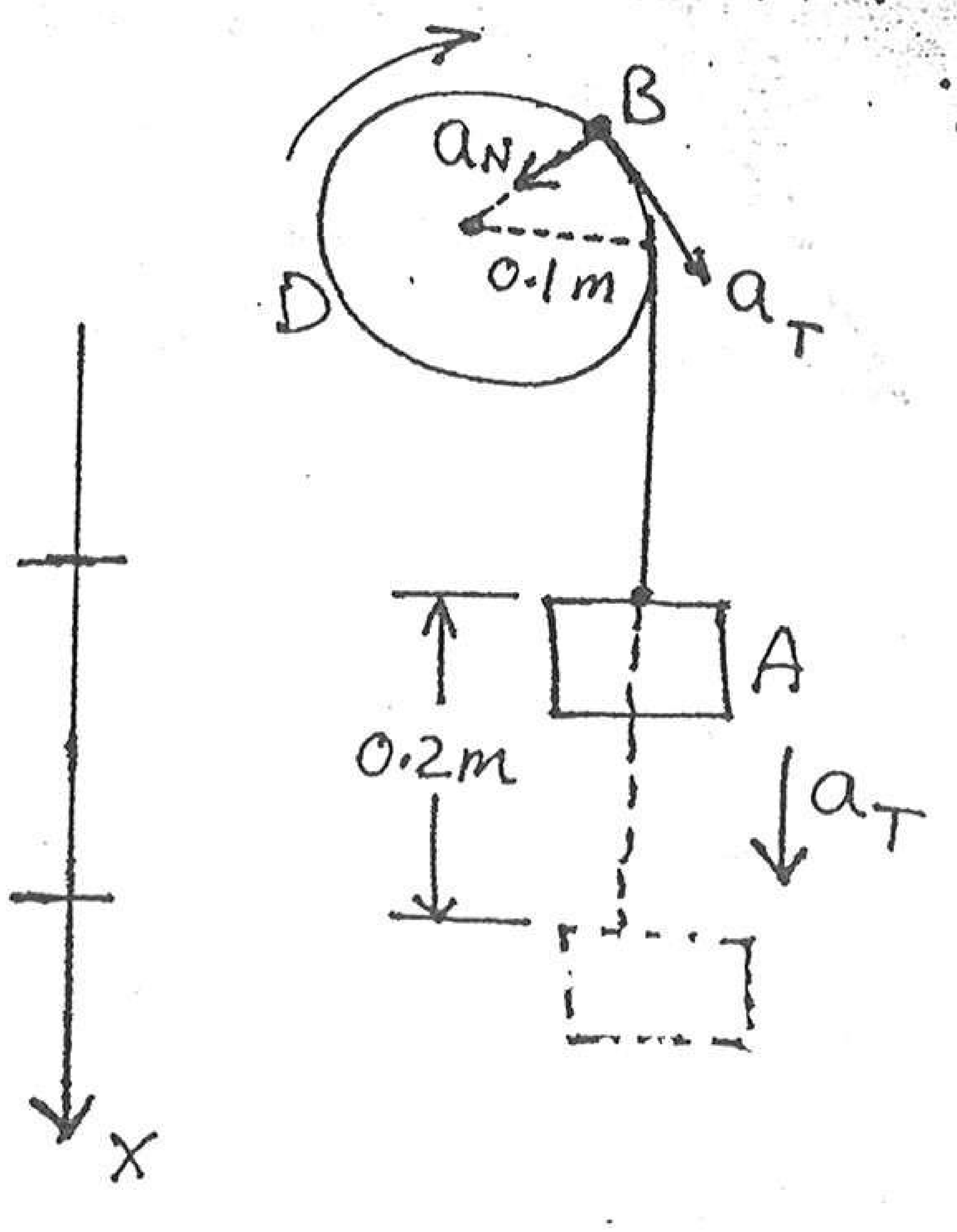
$$a_T = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha$$

total acceleration : $\vec{a} = \vec{a}_N + \vec{a}_T$

* Summary of relations for Circular motion:

Uniform Circular motion	$\alpha = 0$ $\omega = $ Constant $\theta = \theta_0 + \omega(t - t_0)$
Uniformly accelerated Circular motion	$\alpha = $ Constant $\omega = \omega_0 + \alpha(t - t_0)$ $\theta = \theta_0 + \omega_0(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$
Relating formula	$v = \omega R$ $a_N = \omega^2 R$ $a_T = \alpha R$

ex: A disk D rotating in fig
The motion of A is uniformly
accelerated. at $t=0$ the
velocity of body A: 0.04 m/sec,
بعد ثانيتين سقط
and 2 sec later A has fallen
0.2 m. Find the a_T an a_N
at any time.



sol. $X = v_o t + \frac{1}{2} a t^2$ for body A.

$v_o = 0.04 \frac{m}{sec} \Rightarrow X = 0.04t + \frac{1}{2} a t^2$

at $t = 2$ sec $\Rightarrow X = 0.2$ m $\quad \therefore 0.2 = 0.04(2) + \frac{1}{2} a (2)^2$

$\therefore a = 0.06$ m/sec^2

$\Rightarrow X = 0.04t + 0.03t^2$ Position at any time of body.

$v = \frac{dx}{dt} = 0.04 + 0.06t$ [this velocity of body and
حافة
velocity of any Point B on the rim of the disk.

$$a_T = \frac{dv}{dt} = 0.06 \frac{m}{sec^2}$$

$$a_N = \frac{v^2}{R} = \frac{(0.04 + 0.06t)^2}{0.1} = 0.016 + 0.048t + 0.036t^2$$

H.W: For a body in rectilinear motion whose acceleration is given by: $a = 32 - 4v$
(the initial conditions are $X=0$ and $v=4$ at $t=0$)
Find v as a function of t, X as a function of t, and X as a function of v.

Other examples:

ex1: A motorist traveling 60 $Km.hr^{-1}$ brakes to a stop in 100 m.

ⓐ Find his acceleration assuming it to be constant.

ⓑ What would his stopping distance be if his acceleration were $a = -g = -9.8 \frac{m}{sec^2}$

ⓒ What is the stopping time in case ⓐ.

Sol. ⓐ $v^2 = v_0^2 + 2ax$, $v_0 = 60 \frac{Km}{hr}$, $v = 0$

$$\Rightarrow a = \frac{-v_0^2}{2x} = \frac{-1}{2(100)} \left(\frac{60 * 10^3}{3600}\right)^2 = -1.39 \frac{m}{sec^2}$$

Note that the speed had to be converted to $\frac{m}{sec}$ to find the acceleration in $\frac{m}{sec^2}$

ⓑ $v^2 = v_0^2 + 2ax$, $v_0 = 60\ Km/hr$, $v_0 = 0$, $a = -9.8\ m/sec^2$

$$x = \frac{-v_0^2}{2a} = \frac{v_0^2}{2g} = \frac{1}{2(9.8)} \left(\frac{60 * 10^3}{3600}\right)^2 = 14.2\ m$$

ⓒ $v = v_0 + at$, $v = 0$, $v_0 = 60 \frac{Km}{hr}$

$a = -1.39$

$$t = \frac{v - v_0}{a} = -\frac{v_0}{a} = \frac{(60 * 10^3/3600)}{1.39} = 12\ sec$$

ex2: A Car Covers 150 m in 5 sec, with Constant acceleration. Its final velocity is 100 $Km.hr^{-1}$

ⓐ What is the acceleration?

ⓑ What is the initial velocity?

Sol.: For constant acceleration

$$X = X_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

X = final position, X_0 = initial position with $X - X_0 = 150m$

v = final velocity = 100 Km/hr = 27.8 m/sec

v_0 = initial velocity, $t = 5$ sec

$v_0 = v - at$ substituting in the first.

$$X - X_0 = (v - at)t + \frac{1}{2}at^2$$

$$X - X_0 = vt + a\left(\frac{t^2}{2} - t^2\right) = vt - \frac{at^2}{2}$$

$$\Rightarrow a = -\frac{2}{t^2}(X - X_0 - vt)$$

$$a = \frac{-2}{(5)^2}\left[150 - 27.8(5)\right] = -0.88 \frac{m}{sec^2}$$

The initial velocity is:

$$v_0 = v - at = 27.8 + 0.88(5) = 32.2 \frac{m}{sec}$$

Ex3: A ball is thrown vertically upward from a 100m high building with an initial velocity of 50 m/sec.

ⓐ How high does it rise?

ⓑ How much time does it take to reach its maximum height

ⓒ How long does it take to return to the top of the building?

ⓓ What is its velocity at this time?

ⓔ How long does it take to reach the ground?

ⓕ What is its velocity when it hits the ground?

Sol.: $y = y_0 + v_0 t - \frac{1}{2} g t^2 = 100 + 50t - \frac{1}{2}(9.8) t^2$

$v = v_0 - gt = 50 - 9.8t$

$v^2 = v_0^2 - 2g(y - y_0) = (50)^2 - 2(9.8)(y - y_0)$

ⓐ $v^2 = 0$ at the topmost point, where:

$$y - y_0 = \frac{v_0^2}{2g} = \frac{(50)^2}{2(9.8)} = 128\,m$$

$y = y_0 + 128 = 228\,m$

the time when the ball reach topmost point: $v = 0$

$\Rightarrow v = v_0 - gt \Rightarrow 50 - 9.8t = 0 \Rightarrow t = 5.1\,sec$

At this time: $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$$h = y_{max} = 100 + 50(5.1) - \frac{1}{2} 9.8 (5.1)^2 = 228\,m$$

ⓑ from ⓐ $t = 5.1\,sec$

ⓒ at this time $y = 100\,m$ or:

$100 = 100 + 50t - \frac{1}{2} 9.8\, t^2$

either $t=0$ or $t=\frac{2(50)}{9.8}=10.2=2(5.1)$

The rise time is equal to fall time.

(d) $v=50-9.8t=50-(9.8)(10.2)=-50\ \frac{m}{sec}$

(same magnitude as initial velocity but opposite direction).

(e) The ground is reached when $y=0$

$$0=y=100+50t-4.9t^2$$

$$t=\frac{-b\pm[b^2-4ac]^{1/2}}{2a}=\frac{-50\pm[50^2-4(-4.9)100]^{1/2}}{-2(4.9)}$$

$t=-1.71$ sec neglected

$t=11.92$ sec

exy: The motion of a particle is given by:

$$x=2+6t^2-3t^4$$

find the Position, velocity and acceleration at $t=2$sec.

sol. $x\big|_{t=2sec}=2+6(2)^2-3(2)^4=-22\ m$

$$v=\frac{dx}{dt}=12t-12t^3 \Rightarrow v\big|_{t=2sec}=12(2)-12(2)^3=-72\ \frac{m}{sec}$$

$$a=\frac{dv}{dt}=12-36t^2 \Rightarrow a\big|_{t=2sec}=12-36(2)^2=-132\ \frac{m}{sec^2}$$